Mathematics of Gambling and the Nature of Randomness
First Year Seminar Proposal

Alvaro Arias
Professor, Department of Mathematics

Introduction. This is a first-year seminar designed with three goals in mind:

1. To use basic logic to express mathematical ideas precisely and formally.
2. To use games such as card games and dice games to introduce beautiful and elementary mathematical ideas and techniques of discrete mathematics and probability.
3. To help students understand the notion of randomness. In particular, to help develop intuition through simulations.

Course Description.

The course will start with an introduction to logic. We will study truth tables, arguments, and proofs. We will use the connectives (not, and, or, if …, then …) to learn basic simulation techniques in Excel.

The course will continue with a rigorous study of *counting*. This requires only arithmetic of whole numbers (5^{th} grade mathematics) but the problems can be difficult and require **critical thinking.** Counting is the basis of finite mathematics and probability, and it is studied by math, CS, and engineering majors starting in the sophomore year. By the end of this section, students should be able to answer questions like

> *In a popular lottery known as "Lotto 6/49", a player marks a card with six different numbers from the integers 1-49 and wins if his or her numbers match six randomly selected such numbers. In how many ways can a player complete a game card so that at least one number matches any of the six selected?*

We will then learn to use Excel to answer probability questions using simulations. For example, the question:

> *What is the probability of rolling 10 dice and obtaining a number between 20 and 30?*

can be computed exactly, in principle, but the answer requires several pages of work. If we don't need the exact answer, it is much easier to simulate the experiment in Excel.

To develop intuition about *randomness* we will read chapters from the book *Fooled by Randomness: The Hidden Role of Chance in the Markets and in Life,* by N. Taleb and we will do Excel simulations on examples suggested by the book. Normally, we are trained to look for order and structure in our lives. In contrast, the fascinating book by Taleb looks for disorder and randomness.

A typical example from this book is the following: If a stock has performed above average during each of the last five years, can we conclude that it is a good stock? The answer depends on the initial number of stocks. It may be a great stock, but it can also be a bad stock with good luck. Suppose that there are 1,000 mediocre stocks that have only a 40% chance of doing better than average every year. After one year, roughly 400 of these stocks will have done better than average. And after 5 years, roughly 10 of these stocks will have done better than average every year. Did we pick up a good one, or did we pick up noise that results from randomness?

In the last part of the seminar we will study common probability distributions and the probability laws from an experimental point of view. Instead of looking at them theoretically, we will conduct several Excel simulations with different distributions to illustrate these laws. For example, we will see that large averages of independent random variables have a distribution that approaches the normal distribution (this is the bell shaped distribution). This is the content of the Central Limit Theorem. But we will also learn to be careful and not use the normal distribution automatically (this is a common mistake in natural sciences, social sciences, and business). Here, once again, we will use the book *Fooled by Randomness* to find examples that illustrate the misuse of this theorem. For instance, N. Taleb explains that most financial models are built on normal distributions, which have very few jumps. On the other hand, in an article with Mandelbrot, a famous mathematician, they explain that jumps cannot be ignored in the financial world. In fact "just 10 training days can represent half of the returns of a decade". They say that by looking at the normal distribution and ignoring the jumps, "they focus on the grass and miss out the (gigantic) trees". We will use Excel simulation experiments to simulate examples of this nature.

Addressing the Goals of Marsico First Year Seminars.

- **Intellectual community.** I've been using cooperative learning for more than 20 years. I lecture at the beginning of the class for 10 or 15 minutes and then I ask students to work problems in groups. The problems go from basic ones to challenging ones, so even the fastest students will have enough problems to work on. I walk around the class and spend some time with each group. This allows us to know each other, and results in lively classes where everyone participates. The format of the first year seminars is ideal for my teaching style. The small class setting, lively environment, will undoubtedly result in an intellectual community of the students in the class.

- **Rigorous academic expectations.** The students will have weekly assignments that need to be well written and understood. Mathematical concepts are built on top of each other and it is simply not possible to discuss advanced topics if the students have not mastered the previous ones. Rigorous academic expectations are necessary. The class will also have several Excel simulations and the students will have to organize the information appropriately and present the results clearly. This will be part of the grade. There will be one midterm exam and a comprehensive final exam.

- **Improvement of skills.** By its nature, this seminar emphasizes **critical thinking** and **quantitative reasoning**. It also addresses **presentation and argument** and **writing,** but at a lower level. Homework needs to be well written and the simulation presentations need to be clear.

- **Strong academic advising.** The First Year Seminars provide an excellent advising opportunity. The environment of the course is such that students and professor get to know each other well. The small class setting and the discussion on the topics allow the professor to know the academic interests of the students and makes advising easier. I plan to meet with the students individually and help them design the academic plan for the first year.

Proposed Syllabus

Course outline:

Weeks 1 and 2: ***Introduction to logic and introduction to simulations in Excel*** (Study basic truth tables, arguments and proofs. Then use the logic connectives, to do basic simulations in Excel)

Weeks 3 and 4: ***Counting and finite probabilities*** (Study the foundations of probability and counting, including permutations and combinations. The principles will be illustrated by card games and dice games).

Week 5, 6, and 7: ***Randomness and Excel experiments*** (Read chapters from the book fooled by randomness and conduct Excel simulations to develop intuition on random events).

Week 8 and 9: ***Common probability distributions and probability laws*** (Experimental approach to distributions and probability laws).

Required Text:

We will not use a textbook for the class. Several of the topics listed in the syllabus are part of courses I teach regularly. Over the years I have developed several worksheets and handouts that cover the topics and that fit the cooperative style class that I teach.

Supplemental textbooks:

1. Edward Packel: *The mathematics of games and gambling*. The Mathematical Association of America, New Mathematical Library, 1981.
2. Nassim N. Taleb, *Fooled by Randomness: The Hidden Role of Chance in the Markets and in Life* John *H.* New York : Thomson/Texere, 2001.
3. Sheldon Ross, *A first course in probability*. Macmillan Co., New York; Collier Macmillan Ltd., London, *1984.*

A note on the literature

The New Mathematical Library (cf. [1]) is a series of books written by professional mathematicians for undergraduate students of all majors. It strives to present mathematics in an introductory yet precise way. We will also use Ross' book [3] for the second part of the class. Taleb's book [2] will help us find disorder and randomness and will serve as a guide for Excel's simulations.

Assignments: We will have weekly assignments, mathematical in nature, for weeks 1, 2, 3, and 4. For weeks 5-9, the class the assignments will consist of Excel simulations, often in groups, of mathematical problems.

Technology: We will use Microsoft Excel for simulations.

Exams: There will be midterm in week 5 and a final

Learning Outcomes:
Please include the following student learning outcomes in your proposed FSEM syllabus and provide examples of the course assignments and activities that will permit you to assess whether students are meeting the following FSEM student learning outcomes:

a) Demonstrate their membership in an intellectual community by meeting rigorous academic expectations through critical reading, discussion, research, and/or writing.

> I've been using cooperative learning for more than 20 years. I lecture at the beginning of the class for 10 or 15 minutes and then I ask students to work problems in groups. The problems go from basic ones to challenging ones, so even the fastest students will have enough problems to work on. I walk around the class and spend some time with each group. This allows us to know each other, and results in lively classes where everyone participates. The format of the first year seminars is ideal for my teaching style. The small class setting, lively environment, will undoubtedly result in an intellectual community of the students in the class.

b) Practice newly acquired skills in an active learning environment where writing, performing, laboratory experiments, quantitative analyses, or other forms of experiential and/or creative activities will shape the goals and activities of the seminar.

> Most of the topics in the class will be new to students. Some will be familiar with basic probability, but probably none of them knows how to use Excel to simulate probability and solve problems. One of the problems we will cover is the "Secretary Problem", (https://en.wikipedia.org/wiki/Secretary_problem) which is remarkable and that can be approached using simulations. The problems will be worked in groups in class using a cooperative method.