Multivariate meta-analysis:

Steps in conducting a multivariate meta-analysis and fitting a multiple regression model on averaged correlation in R

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Overview

- Background of the problem
  - Typical study
  - Multivariate meta-analysis
- Purpose
- Method
- Analysis
  - Multivariate meta
  - Univariate meta
- Conclusions

Typical study

- Graduate record examination (GRE) is an admission requirement for graduate students
- The predictive validity of the GRE on Graduate student's GPA (GGPA) has been studied extensively by researchers (e.g., Kuncel, Hezlett & Ones, 2001)

Typical study

- GRE-Verbal and GRE-Quantitative are studied as independent outcomes
- Both scores (Verbal & Quantitative) are collected from the same individual
- This leads to the problem of Dependency between the multiple outcomes

Multivariate meta-analysis

- Cheung & Chan (2005): Meta-Analytic SEM approach (MASEM) based on SEM.
  - Based on the estimation of SEM for multiple groups (challenges due to assumptions difficult to meet)
- Becker (1992, 2009): Generalized Least Squares approach.
  - Estimation is much more straightforward. Furthermore, there is a package in R (mvmeta) which estimates it

Multivariate meta-analysis (cont)

Reasons why we should use it:

- Estimate simultaneously more than one effect size
- Interaction between the effect sizes can be studied
- Accounts for dependencies between the effect sizes

Purpose of the study

- Use multivariate meta-analysis to estimate the average effect sizes of the relationship between GRE (Verbal, Quant) and GGPA
- Compare against univariate estimates

Method

- We collected the relationship (correlation) between:
  1. GRE-V and GGPA
  2. GRE-Q and GGPA
  3. GRE-Q and GRE-V
- 13 studies, total of 39 (13 x 3) effect sizes was collected

Data analysis

- Phase 1: Estimate the effect sizes for GGPA-GREQ, GGPA, GREV, GREQ-GREV
  - Fit a multivariate random effects model
- Phase 2: Estimate the effect size for the three variables using univariate meta-analysis
  - Random effects models
- Comparison between the two approaches

Phase 1: Effect size estimates using multivariate approach

```
Random <- mvmeta(cbind(Zr13, Zr23, Zr12),S=S,data=Data, method="reml")
print(summary(Random),digits=3)

## Call:  mvmeta(formula = cbind(Zr13, Zr23, Zr12) ~ 1, S = S, data = Data,
##     method = "reml")
##
## Multivariate random-effects meta-analysis
## Dimension: 3
## Estimation method: REML
##
## Fixed-effects coefficients
##       Estimate  Std. Error       z  Pr(>|z|)  95%ci.lb  95%ci.ub
## Zr13     0.193       0.048   4.051     0.000     0.100     0.287  ***
## Zr23     0.295       0.023  12.833     0.000     0.250     0.340  ***
## Zr12     0.184       0.031   5.889     0.000     0.123     0.245  ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Between-study random-effects (co)variance components
##  Structure: General positive-definite
##       Std. Dev   Corr
## Zr13     0.148   Zr13   Zr23
## Zr23     0.047  0.296
## Zr12     0.083  0.515  0.488
##
## Multivariate Cochran Q-test for heterogeneity:
## Q = 128.843 (df = 36), p-value = 0.000
## I-square statistic = 72.1%
##
## 13 studies, 39 observations, 3 fixed and 6 random-effects parameters
##  logLik      AIC      BIC
##  20.177  -22.353   -8.102
```

Random effect model

Zr13 = Verbal ~GPA
Zr23 = Quant ~GPA
Zr12 = Verbal ~Quant

Phase 2: Effect size estimates using univariate approach

```
#### Model 2: Univariate
## r13 (Verbal ~ GPA)
dat <- escalc(measure="COR", ni=n, ri=r13, data=Data)
UR13<- rma(yi, vi, data=dat)
UR13

## Model Results:
## estimate      se     zval      pval    ci.lb    ci.ub
## 0.1924    0.0439   4.3877   <.0001   0.1065   0.2783      ***
##
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## r23 (Quantitative ~ GPA)
dat2 <- escalc(measure="COR", ni=n, ri=r23, data=Data)
UR23<- rma(yi, vi, data=dat2)
UR23

## Model Results:
## estimate      se     zval      pval    ci.lb    ci.ub
## 0.2962    0.0224  13.1969   <.0001   0.2522   0.3402      ***
##
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## r12 (Verbal ~ Quantitative)
dat3 <- escalc(measure="COR", ni=n, ri=r12, data=Data)
UR12<- rma(yi, vi, data=dat3)
UR12

## Model Results:
## estimate      se     zval      pval    ci.lb    ci.ub
## 0.1886    0.0290   6.5117   <.0001   0.1318   0.2454      ***
##
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Verbal ~GGPA: 0.192

Quant ~GGPA: 0.296

Verbal ~Quant: 0.189

Results: Multivariate vs univariate

Relationship	Approach	
	Univariate	Multivariate
Verbal ~ GGPA	0.192	0.193
Quantitative ~ GGPA	0.296	0.295
Verbal ~ Quantitative	0.189	0.184

The averaged effect sizes are the same between the 2 approaches

An alternative way to approach the problem

- Use the average effect size in a multiple regression model to predict the relationship between GRE Verbal, Quantitative and GGPA

Multiple regression

- GGPA = Verbal + Quantitative
- Estimate the unique prediction of each independent variable (Verbal , Quantitative) on GGPA
- Estimate the overall variance explained by the model (R^2)
- Used matrix equations to answer the question

Results: Multiple regression

```
##### Multiple regression: Averaged correlations from RANDOM effect

##--Step 1: Extract the coefficients from RANDOM effect model
Random$coefficients
##                 Zr13      Zr23      Zr12
## (Intercept) 0.1933438 0.2948303 0.1840404
V_GPA <-Random$coefficients[1]
Q_GPA <-Random$coefficients[2]
V_Q   <-Random$coefficients[3]

##--Step 2: Transfer the extracted values into 2 matrices
####--Matrix Rii (Averaged correlation between the independent variables)
Rii <-matrix(c(1, V_Q, V_Q, 1), nrow=2,ncol=2)
Rii
##             [,1]      [,2]
## [1,] 1.0000000 0.1840404
## [2,] 0.1840404 1.0000000
###--Create an inverse matrix of Rii
###### Note: matrix.inverse function only works for a square matrix
invRii <- matrix.inverse(Rii)
invRii
##              [,1]       [,2]
## [1,]  1.0350583 -0.1904926
## [2,] -0.1904926  1.0350583
###--Matrix Riy (Averaged correlation between the independent and dependent variab
les)
Riy <-matrix(c(V_GPA, Q_GPA), nrow=2, ncol=1)
Riy
##             [,1]
## [1,] 0.1933438
## [2,] 0.2948303
##--Step 3: Compute the coefficients (beta)
B <-invRii%*%Riy
dimnames(B) <-list(c("Verbal", "Quant"), c("Coefficient"))
###### Prints the coefficients with label
print(B)
##        Coefficient
## Verbal   0.1439591
## Quant    0.2683360
###--Step 4: Estimate the R2
#### Transpose Riy matrix
TRiy <-t(Riy)
TRiy
##             [,1]      [,2]
## [1,] 0.1933438 0.2948303
#### Multiply the transposed Riy (TRiy) matrix with beta (B) matrix

R2 <-((TRiy%*%B)*100)
dimnames(R2) <-list(c("R_squared"))
###### Prints the R-squared value with label
print(R2)
##               [,1]
## R_squared 10.69472
```

Steps created manually to fit a regression model using matrix algebra

Unique contribution:
Verbal = 0.144
Quant = 0.268

Total variance explained by the model (R^2) = 10.69%

Conclusions

- The average effect size is the SAME for univariate and multivariate meta-analysis approach
- The level of dependency between the outcomes appeared to have no impact on the effect size
  - What level of correlation among variables (e.g., correlations) is need before dependency becomes an issue?
- Card (2012): relatively new techniques, … "many unresolved issues remain" (p. 279).

Conclusions (cont)

- Fitting a regression model helps to provide unique contribution of the each predictor

Future studies

- Explore the effect of different level of correlation using:
  - Different sample sizes (small, medium, large)
- Simulations

References

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Questions?

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