REVIEW

Early Childhood Mathematics Intervention

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Preschool and primary grade children have the capacity to learn substantial mathematics, but many children lack opportunities to do so. Too many children not only start behind their more advantaged peers, but also begin a negative trajectory in mathematics. Interventions designed to facilitate their mathematical learning during ages 3 to 5 years have a strong positive effect on these children's lives for many years thereafter.

Very young children have the potential to learn mathematics that is complex and sophisticated (*1*, *2*). Unfortunately, this potential is left unrealized for many children throughout the world (*1*–*5*). Fortunately, research-based early childhood mathematics interventions exist that increase these children's mathematical knowledge (*6*). There is much to gain, and little to lose, by engaging young children in mathematical experiences.

Mathematical thinking is cognitively foundational. Preschool children's knowledge of mathematics predicts their later school success into elementary (*7*) and even high school (*1*, *6*). Further, it predicts later reading achievement even better than early reading skills (*7*), and the study of mathematics in high school predicts college science achievement across subjects (*8*). The quantitative, spatial, and logical reasoning competencies of mathematics may form a cognitive foundation for thinking and learning across subjects. Given the importance of mathematics to academic success and to a nation's economic success (*6*, *9*), all children need a robust knowledge of mathematics in their earliest years.

Not all children have adequate opportunities to develop this cognitive foundation. For example, some 6-year-olds have not acquired mathematical knowledge that other children acquire at 3 years of age (*5*). Although both groups of children may have informal experiences with quantitative situations, those from low-resource communities may have fewer opportunities to mathematize this tacit knowledge; that is, to reflect on and represent the situations [with cognitive tools, from verbal language to finger patterns; compare (*9*)]. For example, children from low- and middle-income families perform similarly on mathematics problems involving physical objects. When shown three counters that are then covered, then shown one more added to those under the cover, children from both groups perform equally well in making a matching set of four. However, children from middle-income groups perform significantly better in solving similar problems presented verbally and without physical objects (*10*). Children from low-income families also are less able to explain mathematical ideas and processes (*10*). Such representations and explanations constitute valued goals in mathematics education (*6*), are essential components of mathematical knowledge (e.g., preverbal number knowledge is shared by other species and is not mathematical until it is represented) (*1, 2*), and play an influential role in promoting future mathematics learning (*11*). Therefore, children must learn to mathematize their informal experiences by abstracting, representing, and elaborating them mathematically. If they do not, they miss the opportunity to learn the language of mathematics in all its multifaceted forms.

High-quality education can help children mathematize (*4*). Without such education beginning in preschool, too many children, especially from low-resource communities, follow a path of failure in mathematics (*1*). However, present-day early childhood classrooms in many countries do not provide high-quality mathematics experiences, with many children learning little over the course of an entire academic year (*1*, *5*, *12*–*14*) and some regressing on certain skills (*15*).

Early childhood teachers often believe they are "doing mathematics" when they provide puzzles, blocks, and songs. Even when they teach mathematics, that content is usually not the main focus, but is embedded in a fine-motor or reading activity. Unfortunately, evidence suggests that such an approach is ineffective, owing to a lack of explicit attention to mathematical concepts and procedures along with a lack of intentionality to engage in mathematical practices (*1*, *16*).

To improve mathematics learning for all young children, and especially to address inequities faced by children from low-resource communities,

Fig. 1. Learning to compose shapes using physical objects and computer manipulatives. This girl is operating at the initial level of thinking. The *Building Blocks* software moves forward or backward along the learning trajectory.

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developers have designed research-based interventions. These positively affect children's competencies in mathematics and beyond.

Scientific Interventions

Several research-based interventions for 3- to 5-year-old children have been scientifically evaluated with positive effects, including *Rightstart* (*4*), *Pre-K Mathematics* (*17*, *18*), and *Building Blocks* (*12*), while others show promise but await rigorous evaluation, such as *Big Math for Little Kids* (*19*). Two of these interventions share several characteristics, allowing the abstraction of general principles guiding effective interventions for preschool children. We first describe the two interventions and their initial empirical support, then describe their shared characteristics.

The authors of the *Rightstart* program theorized that children separately build initial counting competencies, intuitive ideas of quantity comparison, and initial notions of change (e.g., a group gets bigger when items are added). The integration of these separate ideas forms a central conceptual structure for number. On this basis, activities were designed to help children build each separate competence and then integrate them. For example, the program used games and experiences with different models of number (e.g., groups of objects, pictures, thermometers, or dials; the program was renamed *Number Worlds* to emphasize this characteristic) to develop children's central conceptual structure for number.

This program improved young children's knowledge of number, which supported their learning of more complex mathematics through first grade (*4*). In a 3-year longitudinal study, children from low-resource communities who experienced the program from kindergarten surpassed both a second low-resource group and a mixed-resource group who showed a higher initial level of performance and attended a magnet school with an enriched mathematics curriculum (*20*, *21*). Although there are caveats, given that the *Number Worlds* teachers received substantial help from the program developers and expert teachers, and the number of students was small (*21*), these results suggest that scientifically based interventions have the potential to close achievement gaps in mathematics.

The second program, *Building Blocks*, was developed and evaluated according to a comprehensive research framework (*22*). *Building Blocks*' basic approach is finding the mathematics in, and developing mathematics from, children's activity. The curriculum was designed to help children extend and mathematize their everyday activities, from building blocks to art and stories to puzzles and games (Fig. 1). Educational goals included developing competence in the two domains consistently identified as foundational: (i) number concepts (including counting and the earlier developing competence of subitizing, or recognizing the numerosity of a group quickly) and arithmetical operations, and (ii) spatial and geometric concepts and processes. Each of these domains was structured along research-based learning trajectories (*1*, *2*), a construct to which we will return. A series of studies documents that *Building Blocks* increases the mathematics knowledge of preschoolers from low-resource communities more than "business-as-usual" curricula [e.g., (*12*)].

Fig. 2. Identifying shapes in a "feely box" before seeing them develops children's ability to identify shapes by their attributes.

The *Number Worlds* and *Building Blocks* programs share several characteristics. Both sets of authors used research to include a comprehensive set of cognitive concepts and processes (*Number Worlds* focused only on the domain of numbers). Both programs use a mix of instructional methods, including explicit instruction (but not overly didactic, which can have negative outcomes for the youngest children) (*23*). Both are based on developmentally sequenced activities, and both help teachers learn about, assess, and intervene on the basis of those sequences. This characteristic is central to the *Building Blocks* curriculum, with every aspect (e.g., text, software, and professional development) connected to an explicit core of learning trajectories for each mathematical topic. Similar use of learning trajectories (often using different terms, but sharing the core construct) in designing curricula and professional development may be responsible for the success of many early mathematics projects (*3*, *12*, *24*, *25*).

Learning trajectories: Directions for successful learning and teaching. On the basis of Simon's seminal work (*26*), we define a learning trajectory as composed of three components: a goal, a developmental progression, and instructional activities (*2*). To attain a certain mathematical competence in a given topic (the goal), children learn each successive level of thinking (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects that enable thinking at each higher level. For example, the goal might be for young children to become competent counters, counting being the first and most basic mathematical algorithm. The developmental progression describes a typical path children follow in developing an understanding of and skill in counting. At one level of thinking, they acquire the cardinality concept by connecting the last number of the counting processes to the output of their subitizing (which by definition is cardinal). This catalyzes a count-to-cardinal transition, producing a cardinal value ("fourness" connected to subitized images) and verbal label ("four") that are associated with the set counted (*1*). (For more detail on this learning trajectory, see the SOM text and fig. S1.)

Such learning trajectories provide not only multiple educational advantages but also a core around which varied educational activities can be structured. The levels of thinking of their developmental progressions integrate the essential aspects of concepts, skills, and problem solving (*6*, *27*) and provide benchmarks for assessments (*1*, *2*, *5*). Research-based instructional activities provide guidelines for writing curriculum, teaching, and professional development. Such guidance is especially important for professional development because early childhood teachers' knowledge of mathematics, young children's mathematical development, and instruction are positively correlated to their children's achievement (*1*, *2*, *28*). Without such knowledge, teachers of young children often offer tasks that are either too easy or too hard for children, and do not

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recognize the mismatch (*29*). Thus, teachers need integrated knowledge of all three components of learning trajectories: the mathematical content (goal), the developmental progressions of children's thinking and learning, and instructional tasks and teaching strategies that help children move along those progressions (*1*). In this way, learning trajectories can facilitate developmentally appropriate teaching and learning for all children (*1*). Other early mathematics projects that demonstrate learning gains share many of these conceptual foundations (*2*, *3*, *30*).

Although promising, initial studies of *Number Worlds* and *Building Blocks* used the individual child as the unit of analysis, despite their assignment to treatments by classroom, which can inflate findings. Therefore, we conducted a larger study involving cluster randomized assignment of 36 classrooms (*24*). The *Building Blocks* curriculum increased the quantity and quality of the mathematics environment and teaching, and substantially increased scores on a mathematics achievement test, with the *Building Blocks* group significantly outperforming both a "business-as-usual" control group and a group using the *Pre-K Mathematics* curriculum, which was not based on the learning trajectories construct.

A subsequent study evaluated whether these results could be scaled up; that is, could an intervention framework be designed that maintained the integrity of practices of the intervention in increasingly wider contexts characterized by increases in both number (of children, teachers, etc.) and complexity (*6*)? Based on a synthesis of literature, we created the TRIAD (Technology-enhanced, Research-based, Instruction, Assessment, and professional Development) framework, whose guidelines include collaboration among key groups (e.g., administrators, teachers, families); extensive, multifaceted professional development; and strategies to maintain the integrity of the research-based *Building Blocks* curriculum. TRIAD emphasizes both teaching for understanding and following learning trajectories (*31*). Over the course of 2 years, teachers participated in 12 full days of professional development with presentations, discussions, and role-playing addressing all three components of the learning trajectories, as well as use of the Web application *Building Blocks Learning Trajectories (BBLT)*, which presented and linked the components. *BBLT* provides scalable access to the learning trajectories by means of descriptions, videos, commentaries, and connections between children's levels of thinking and instruction (*25, 31*) (see also UBTRIAD.org). Teachers then implemented the curriculum with mentoring based on an observational fidelity instrument.

The experiment supported the efficacy of the TRIAD model, with strong positive effects on children's achievement (*25*). Most groups (e.g., girls and boys, different compositions of socioeconomic status) demonstrated equal learning gains. However, African American children in the control group showed smaller gains than their peers in the same group. Inversely, within the TRIAD group African American children showed larger gains than their peers (narrowing, but not closing the initial achievement gap). The TRIAD/*Building Blocks* intervention may be particularly effective in ameliorating the negative effects of some educators' low expectations for African American children's learning of mathematics (*6*) through providing learning trajectories that help teachers see what children can do and how they can be helped to progress to higher levels of mathematical thinking.

Several effective primary grades interventions also use some version of the learning trajectories construct (*3*, *4*, *16*, *32*, *33*), for example, emphasizing the use of research-based progressions in formative assessment (*6*). They explicitly attend to conceptual understanding by addressing, discussing, and developing connections among concepts, facts, procedures, and processes (*34*) and do not drill basic facts until conceptual foundations and meaningful strategies are developed (*35*). They challenge students to solve demanding mathematical problems, going beyond learning facts, helping them to learn to think mathematically (*27*).

Conclusion

Mathematics is cognitively foundational, with early mathematics competence a strong predictor of later school success. Young children have the potential to learn mathematics that is both deep and broad. For many, especially those from low-resource communities, this potential has been unrealized. Structured, research-based mathematics interventions have shown to be effective in helping all children learn mathematics (Fig. 2). Evidence supports interventions that provide foundational and mathematical experiences in number, space, geometry, measurement, and the processes of mathematical thinking.

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Supporting Online Material

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Figs. S1 and S2
References

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