PHYS 1211 *University Physics I* — Problem Set 3

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Instructor

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And All That...

My favorite method of computing the vector product is the determinant method discussed in HRW Appendix E. Here we write that:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix},$$

where for example $\vec{a} = a_x\hat{i} + a_x\hat{j} + a_x\hat{k}$.

Answer the following (20 points total):

1. **Show that evaluating this determinant gives an expression equivalent to Eq. 3-30 of HRW.**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \left(a_y b_z\ \hat{i} + a_z b_x\ \hat{j} + a_x b_y\ \hat{k}\right) - \left(a_z b_y\ \hat{i} + a_x b_z\ \hat{j} + a_y b_x\ \hat{k}\right)$$

Now collect all the $\hat{i}$, $\hat{j}$, and $\hat{k}$ terms:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\ \hat{i} + (a_z b_x - a_x b_z)\ \hat{j} + (a_x b_y - a_y b_x)\ \hat{k}$$

Which is equivalent to HRW Eq. 3-30.

Now we consider a special vector called "del," written $\vec{\nabla}$. This vector will seem like a strange beast, because instead of numbers (such as those represented by a_x, and b_x and so on) this vector has *derivatives*:

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$

Here the use of ∂ instead of a regular d means that the derivative is taken *only* with respect to the indicated variable, for example:

$$\frac{\partial}{\partial x}xy^2 = y^2, \qquad \frac{\partial}{\partial y}xy^2 = 2xy, \qquad \frac{\partial}{\partial z}xy^2 = 0.$$

2. **Write the vector product $\vec{\nabla} \times \vec{E}$ as a determinant.**

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

3. **If $\vec{E} = x\hat{i} + y\hat{j} + z\hat{k}$, what is $\vec{\nabla} \times \vec{E}$?**

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \left(\frac{\partial}{\partial y} z\ \hat{i} + \frac{\partial}{\partial z} x\ \hat{j} + \frac{\partial}{\partial x} y\ \hat{k} \right) - \left(\frac{\partial}{\partial z} y\ \hat{i} + \frac{\partial}{\partial x} z\ \hat{j} + \frac{\partial}{\partial y} x\ \hat{k} \right) = 0$$

All of the partial derivatives give 0.

4. **For the same $\vec{E}$, what is $\vec{\nabla} \cdot \vec{E}$?**

$$\vec{\nabla} \cdot \vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \left(x\hat{i} + y\hat{j} + z\hat{k} \right) = \left(\frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z \right) = 3$$

HRW Chapter 3 Problems

1. A vector $\vec{a}$ can be represented in the *magnitude*-*angle* notation (a, θ), where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

is the angle $\vec{a}$ makes with the positive x axis.

(a) Given $A_x = -25.0$ m and $A_y = 40.0$ m, $A = \sqrt{(-25.0\text{ m})^2 + (40.0\text{ m})^2} = 47.2\text{ m}$

(b) Recalling that tan θ = tan (θ + 180°), $\tan^{-1}$ [(40.0 m)/ (– 25.0 m)] = – 58° or 122°. Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility.

2. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a)

$$20.0° = (20.0°)\frac{2\pi\text{ rad}}{360°} = 0.349\text{ rad}.$$

(b)

$$50.0° = (50.0°)\frac{2\pi\text{ rad}}{360°} = 0.873\text{ rad}.$$

(c)

$$100° = (100°)\frac{2\pi\text{ rad}}{360°} = 1.75\text{ rad}.$$

(d)

$$0.330\text{ rad} = (0.330\text{ rad})\frac{360°}{2\pi\text{ rad}} = 18.9°.$$

(e)

$$2.10\text{ rad} = (2.10\text{ rad})\frac{360°}{2\pi\text{ rad}} = 120°.$$

(f)

$$7.70\text{ rad} = (7.70\text{ rad})\frac{360°}{2\pi\text{ rad}} = 441°.$$

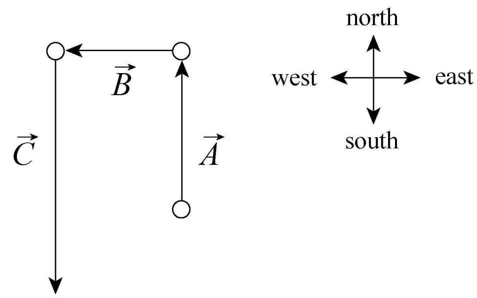
6. (a) With r = 15 m and θ = 30°, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15 \text{ m}) \cos 30° = 13 \text{ m}.$$

(b) Similarly, the y component is given by $r_y = r \sin\theta = (15 \text{ m}) \sin 30° = 7.5$ m.

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\text{i}}$ direction (+x direction) and *north* as the $\hat{\text{j}}$ direction (+y direction) All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown below:



$$\vec{A} = (3.1 \text{ km})\,\hat{\text{j}}$$
$$\vec{B} = (-2.4 \text{ km})\,\hat{\text{i}}$$
$$\vec{C} = (-5.2 \text{ km})\,\hat{\text{j}}$$

(b) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (-2.4 \text{ km})\hat{\text{i}} + (-2.1 \text{ km})\hat{\text{j}}$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4 \text{ km})^2 + (-2.1 \text{ km})^2} \approx 3.2 \text{ km} .$$

(c) There are two possibilities for the angle:

$$\theta = \tan^{-1}\left(\frac{-2.1 \text{ km}}{-2.4 \text{ km}}\right) = 41°, \text{or } 221° .$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +x axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

12. (a) $\vec{a}+\vec{b}=(3.0\,\hat{\mathrm{i}}+4.0\,\hat{\mathrm{j}})\ \mathrm{m}+(5.0\,\hat{\mathrm{i}}-2.0\,\hat{\mathrm{j}})\ \mathrm{m}=(8.0\ \mathrm{m})\,\hat{\mathrm{i}}+(2.0\ \mathrm{m})\,\hat{\mathrm{j}}$.

(b) The magnitude of $\vec{a}+\vec{b}$ is

$$|\vec{a}+\vec{b}|=\sqrt{(8.0\ \mathrm{m})^2+(2.0\ \mathrm{m})^2}=8.2\ \mathrm{m}.$$

(c) The angle between this vector and the $+x$ axis is $\tan^{-1}[(2.0\ \mathrm{m})/(8.0\ \mathrm{m})] = 14°$.

(d) $\vec{b}-\vec{a}=(5.0\,\hat{\mathrm{i}}-2.0\,\hat{\mathrm{j}})\ \mathrm{m}-(3.0\,\hat{\mathrm{i}}+4.0\,\hat{\mathrm{j}})\ \mathrm{m}=(2.0\ \mathrm{m})\,\hat{\mathrm{i}}-(6.0\ \mathrm{m})\,\hat{\mathrm{j}}$.

(e) The magnitude of the difference vector $\vec{b}-\vec{a}$ is

$$|\vec{b}-\vec{a}|=\sqrt{(2.0\ \mathrm{m})^2+(-6.0\ \mathrm{m})^2}=6.3\ \mathrm{m}.$$

(f) The angle between this vector and the $+x$ axis is $\tan^{-1}[(-6.0\ \mathrm{m})/(2.0\ \mathrm{m})] = -72°$. The vector is 72° *clockwise* from the axis defined by $\hat{\mathrm{i}}$.

33. (a) The scalar (dot) product is (4.50)(7.30)cos(320º – 85.0º) = – 18.8 .

(b) The vector (cross) product is in the $\hat{\mathrm{k}}$ direction (by the right-hand rule) with magnitude |(4.50)(7.30) sin(320º – 85.0º)| = 26.9 .

41. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B}=AB\cos\theta$, we have

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A=6.00$, $B=7.00$ and $\vec{A}\cdot\vec{B}=14.0$, $\cos\theta=0.333$, or $\theta=70.5°$.

45. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

(a) The magnitude of the net displacement is

$$|\vec{AB}|=\sqrt{|AD|^2+|AC|^2}=\sqrt{(17.0\ \mathrm{m})^2+(22.0\ \mathrm{m})^2}=27.8\mathrm{m}.$$

(b) The magnitude of the vertical component of $\vec{AB}$ is $|AD|\sin 52.0° = 13.4$ m.