PHYS 1211 *University Physics I* — Problem Set 2

Winter Quarter, 2007 — SOLUTIONS

Instructor

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The Flying Train

The world record speed for a train (581 km/h) was set recently by a Japanese Magnetically-levitated, or Mag-Lev Train. Several other countries are researching or already using these "flying trains" for high-speed rail travel. They use the repulsive force between large magnets (sometimes electromagnets made from high-temperature superconducting materials) to float the train off a special track. As a result the vehicle can move with no friction. We will discuss friction and magnetic forces later in the course. For now, let's examine the kinematics of these trains.

One developer of this technology states that, from a standing start, its train reaches 300 km/h in a distance of only 5 km. A current state-of-the-art high speed train (for example the German ICE) requires 30 km to reach the same speed.

Answer the following (30 points total):

1. **How fast is 300 km/h in miles-per-hour? How about in meters per second?[1 point]**

   Simple unit conversions:

   $$300\ \frac{\text{km}}{\text{h}} \cdot \frac{0.621\ \text{mi}}{\text{km}} = 186\ \frac{\text{mi}}{\text{h}}$$

   and

   $$300\ \frac{\text{km}}{\text{h}} \cdot \frac{1\ \text{h}}{3600\ \text{s}} \cdot \frac{1000\ \text{m}}{1\ \text{km}} = 83.3\ \frac{\text{m}}{\text{s}}$$

2. **Calculate the acceleration of each train, assuming it remains constant until 300 km/h is reached. [3 points]**

   Here we know initial and final velocities, and initial and final positions: $v_0 = 0$ m/s, $v_{max} = 83.3$ m/s, $x_0 = 0$ m, $x = 5000$ m. Use:

   $$v^2 = v_0^2 + 2a\,(x - x_0)$$

   Solve for a:

   $$a = \frac{v^2 - v_0^2}{2\,(x - x_0)}$$

   For the Mag-Lev train,

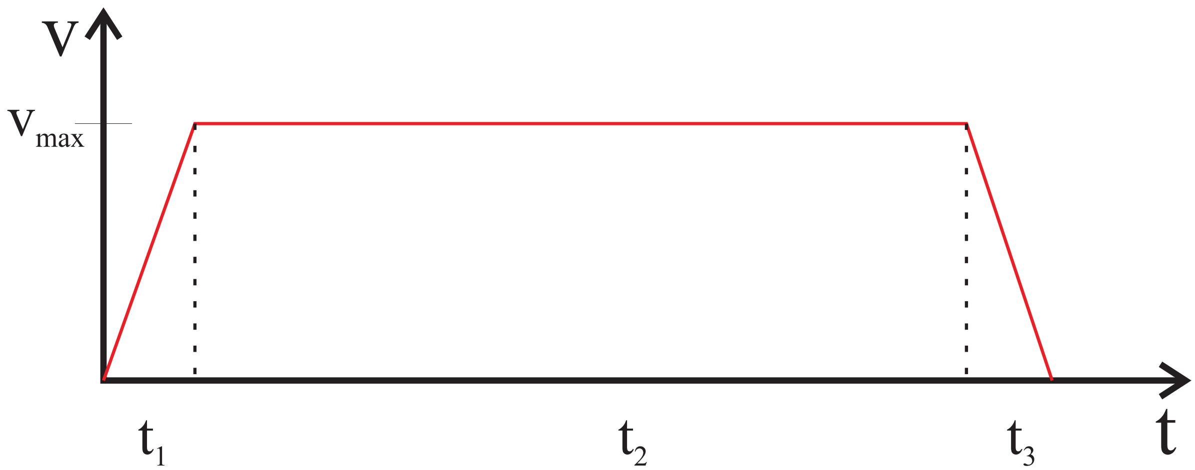
   $$a_{\text{ML}} = \frac{(83.3\ \text{m/s})^2 - (0\ \text{m/s})^2}{2\,(5000\ \text{m} - 0\ \text{m})} = 0.694\ \frac{\text{m}}{\text{s}^2}$$

and for the ICE,

$$a_{\text{ICE}} = \frac{(83.3 \text{ m/s})^2 - (0 \text{ m/s})^2}{2\,(30,000 \text{ m} - 0 \text{ m})} = 0.116 \ \frac{\text{m}}{\text{s}^2}$$

3. **Let's pretend RTD is considering installing a high-speed train between Denver and Boulder, and have hired you to consult on the choice between a Mag-Lev and an ICE-type train. Calculate the time of the Denver-Boulder express trip (no stops) in the Mag-Lev train, assuming the route is 30 miles long, and that the acceleration is the same for starting and stopping the train. [5 points]**

   First, to understand this a bit better sketch the velocity of the ML train as a function of time:



We have constant acceleration, a_{ML}, which means linearly increasing velocity, up to the maximum speed of 83.3 m/s. This takes some period of time, t_1. Then we travel along at top speed for some time, t_2. Then we have constant (negative) acceleration back to zero velocity, which takes some time $t_3 = t_1$. Now we calculate these times separately. Because *velocity* is constant during time t_2, and we know that the distance covered during this time is the total length of the trip (30 miles = 48 km) minus the 10 km we are accelerating and decelerating, we simply use the definition of velocity to determine

$$t_2 = \frac{\text{distance}}{v_{\text{max}}} = \frac{38,000 \text{ m}}{83.3 \text{ m/s}} = 456 \text{ s} = 7.6 \text{ minutes}$$

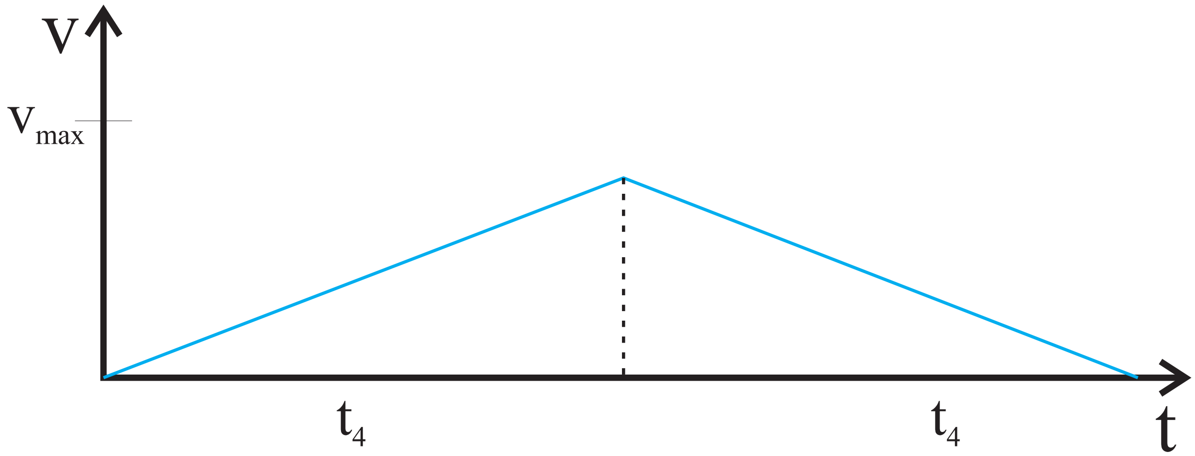
Now we get t_1 from the constant acceleration equations,

$$v = v_0 + at \rightarrow t_1 = \frac{v - v_0}{a} = \frac{83.3 \text{ m/s} - 0 \text{ m/s}}{0.69 \text{ m/s}^2} = 120 \text{ s} = 2 \text{ minutes}$$

So our total trip time in the ML train is $7.6 \text{ min} + 2\,(2 \text{ min}) = 11.6 \text{ min}$

4. **Now determine the time for the trip in the ICE, but remember this wheeled train needs 30 km to reach top speed, and 30 km to stop from top speed. You will find that puts the train well beyond Boulder. To stop the train in Boulder where should you start slowing down? What is the highest speed the ICE achieved? How long is the trip in the ICE? [5 points]**

   Again start with a plot. Remember we have to be at zero velocity in Boulder! To reach top speed would take 30 km, and then to reach zero another 30 km, putting our passengers well north of Boulder!



In the low acceleration ICE train we have constant acceleration for half the distance, then constant decceleration for the remaining half. The first half of the trip takes:

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\,(24,000 \text{ m})}{0.12 \text{ m/s}^2}} = 645 \text{ s} = 10.8 \text{ minutes}$$

So the total travel time in the ICE is 22 minutes. We can calculate the maximum speed from:

$$v = v_0 + at \rightarrow v = (0 \text{ m/s}) + (0.126 \text{ m/s}^2) \cdot (645 \text{ s}) = 74.6 \text{ m/s} = 269 \text{ km/h}$$

5. **Now consider a non-express route that stops in Westminster (10 miles), Broomfield (20 miles), and Superior (25 miles) before reaching Boulder. What is the total time of the non-express Denver-Boulder trip for each train? [10 points]**

This trip breaks into two 10 mile (16.1 km) legs, Denver-Westminster and Westminster-Broomfield, and two 5 mile (8.05 km) legs, Broomfield-Superior and Superior-Boulder. We calculate the time for each leg, for each train using the same method as above.

For the ML train, each 16.1 km leg breaks into 5 km of acceleration, 6.1 km of constant 300 kph travel, and 5 km of deceleration. We already know $t_1 = 120$ s $= 2$ min is the time for acceleration (or deceleration). The travel time for 6.1 km at 300 kph is then $t_5 = (6.1 \text{ km})/(300 \text{ km/h}) = 1.22$ min. Each 16.1 km leg then takes 5.22 minutes.

On the short 8.05 km leg, even the ML train can't reach top speed. We use the same math here as in number 4. The time for acceleration (or deceleration) is:

$$t_6 = \sqrt{\frac{2x}{a}} = \sqrt{\frac{8.05 \times 10^3 \text{ m}}{0.694 \text{ m/s}^2}} = 108 \text{ s}$$

So the total time for the short leg in the ML train is $2t_6 = 216$ s $= 3.6$ min. The total time for 2 long legs and 2 short legs is $2 \times (5.22 \text{ min}) + 2 \times (3.6 \text{ min}) = 17.6$ min.

For the ICE train, we again never reach top speed. For the long leg the time for acceleration (or deceleration) is :

$$t_7 = \sqrt{\frac{2x}{a}} = \sqrt{\frac{16.1 \times 10^3 \text{ m}}{0.116 \text{ m/s}^2}} = 373 \text{ s}$$

and for the short leg:

$$t_8 = \sqrt{\frac{2x}{a}} = \sqrt{\frac{8.05 \times 10^3 \text{ m}}{0.116 \text{ m/s}^2}} = 263 \text{ s}$$

So that the total non-express route takes $2 \cdot (2t_7) + 2 \cdot (2t_8) = 2540$ s $= 42$ min

6. **For comparison, how long do the current Express and non-express RTD buses take to travel from Denver to Boulder? (Route BX and B, www.rtd-denver.com) [1 point]**

   Currently, the express bus takes between 30 and 40 minutes and the non-express 60 minutes or more.

7. **What is your final recommendation to RTD? Consider that the Mag-Lev is slightly more costly than the ICE, but either one is considerably more expensive than the bus system. [5 points]**

   My final recommendation would be to either spend a little more and get the ML system, or stick with the buses, perhaps making a smaller investment to add more buses and add or upgrade HOV lanes.

HRW Chapter 2 Problems

1. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\, t_c$.

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time interval is

$$t_1 = \frac{(40 \text{ km})}{(30 \text{ km/h})} = 1.33 \text{ h}.$$

During the second part the displacement is Δx_2 = 40 km and the time interval is

$$t_2 = \frac{(40 \text{ km})}{(60 \text{ km/h})} = 0.67 \text{ h}.$$

Both displacements are in the same direction, so the total displacement is

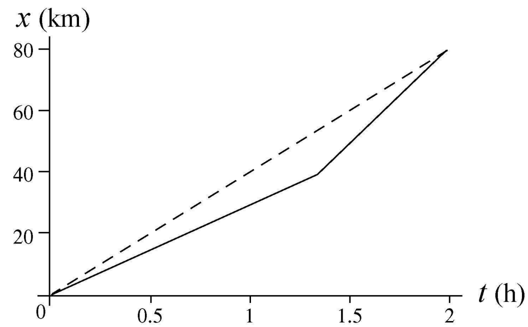
$$\Delta x = \Delta x_1 + \Delta x_2 = 40 \text{ km} + 40 \text{ km} = 80 \text{ km}.$$

The total time for the trip is $t = t_1 + t_2 = 2.00$ h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{(80 \text{ km})}{(2.0 \text{ h})} = 40 \text{ km/h}.$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to (t_1, x_1) = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting (t_1, x_1) to (t, x) = (2.00 h, 80 km). From the graphical point of view, the slope of the dashed line drawn from the origin to (t, x) represents the average velocity.



2. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

3. The speed (assumed constant) is v = (90 km/h)(1000 m/km) ⁄ (3600 s/h) = 25 m/s. Thus, in 0.50 s, the car travels (0.50 s)(25 m/s) ≈ 13 m.

5. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write

$$x = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3.$$

We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in t = 1 s yields x = 3 – 4 + 1 = 0.

(b) With t = 2 s we get $x = 3(2) - 4(2)^2 + (2)^3 = -2$ m.

(c) With t = 3 s we have x = 0 m.

(d) Plugging in t = 4 s gives x = 12 m.

For later reference, we also note that the position at t = 0 is x = 0.

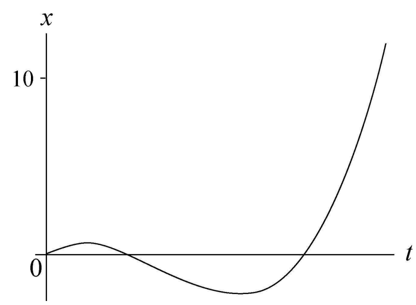
(e) The position at t = 0 is subtracted from the position at t = 4 s to find the displacement Δx = 12 m.

(f) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement Δx = 14 m. Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m/s}.$$

(g) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2, -2)$ to the highest point shown (at t = 4 s) which would represent the answer for part (f).



18. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

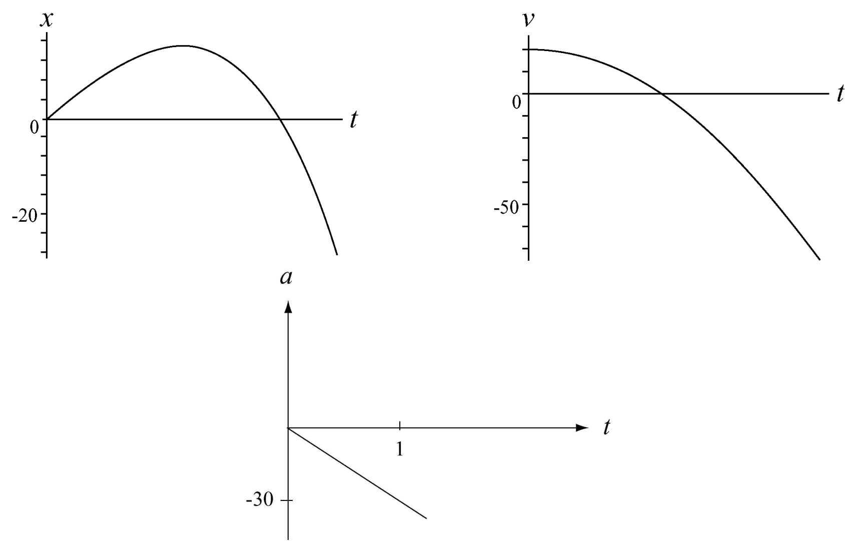
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2\text{ s}$.

(b) From 0 = – 30t, we find $a(0) = 0$ (that is, it vanishes at t = 0).

(c) It is clear that $a(t)$ = – 30t is negative for $t > 0$

(d) The acceleration $a(t)$ = – 30t is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



25. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}(3.0 \times 10^8 \text{ m/s})}{9.8 \text{ m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)(3.1 \times 10^6 \text{s})^2 = 4.6 \times 10^{13} \text{ m} .$$

35. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a:

$$0.0 - (-2.0 \text{ m}) = v_0 (1.0 \text{ s}) + \frac{1}{2} a (1.0 \text{ s})^2$$

$$6.0 \text{ m} - (-2.0 \text{ m}) = v_0 (2.0 \text{ s}) + \frac{1}{2} a (2.0 \text{ s})^2 .$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0$ m/s^2.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.