FORECASTING IN DYNAMIC FACTOR MODELS SUBJECT TO STRUCTURAL INSTABILITY

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1. Introduction

An ongoing theme in David Hendry's work has been concern about detecting and avoiding forecast breakdowns that arise because of structural instability. Parameter instability can arise for various reasons, including structural breaks in the economy (for example, changes in technology), policy regime shifts, or changes in the survey instruments from which the time series are constructed. Hendry and coauthors have argued that such instability, whatever its source, often manifests itself as breaks in time series forecasting relations, and moreover that such breaks constitute one of the primary reasons for forecast failures in practice (see for example Clements and Hendry [1999, 2002], Hendry and Clements [2002], Hendry [2005], and Hendry and Mizon [2005]). One line of Hendry's research has been to develop and to analyze non-structural forecasting methods for their potential robustness to parameter instability, including error correction models, overdifferencing, intercept shift methods, and – closest to the focus of this paper – forecast pooling (Hendry and Clements [2002]).

This paper continues this line of inquiry, in which forecasting methods are examined for their reliability in the face of structural breaks. We focus here on forecasts constructed using dynamic factor models (DFMs; Geweke [1977], Sargent and Sims [1977]). In DFMs, the comovements of the observable time series are characterized by latent dynamic factors. Over the past decade, work on DFMs has focused on highdimensional systems in which very many series depend on a handful of factors (Forni, Lippi, Hallin, and Reichlin [2000], Stock and Watson [2002a, 2002b], and many others; for a survey, see Stock and Watson [2006]). These factor-based forecasts have had notable empirical forecasting successes. Yet, there has been little work to date on the performance of factor-based macroeconomic forecasts under structural instability (exceptions are Stock and Watson (1998, 2002b) and Banerjee, Marcellino, and Masten (2007), which are discussed below).

Despite the limited research on the effect of structural instability on forecasting using factor models, it is plausible that factor-based forecasts might be robust to certain types of structural instability, for reasons akin to those discussed in Hendry and Clements (2002) in the context of forecast pooling. Hendry and Clements (2002) consider forecast

breakdowns arising from intercept shifts, which in turn arise from shifts in the means of omitted variables. These intercept breaks doom the forecasting regression in which they arise, but if one averages forecasts over many forecasting regressions, and if the intercept shifts are sufficiently uncorrelated across the different regressions, then the intercept shifts average out and the pooled forecast is relatively more robust to this source of structural instability than any of the constituent forecasting regressions. In factor models, a similar logic could apply: even if factor loadings are unstable, if the instability is sufficiently independent across series then using many series to estimate the factors could play the same "averaging" role as the pooling of forecasts, and the estimated factors could be well estimated even if individual relations between the observable series and the factors are unstable. Given well-estimated factors, forecasts can be made by standard time-varying parameter or rolling regression methods.

This paper provides empirical results concerning the estimation of dynamic factors and their use for forecasting when there is structural instability in the underlying factor model. Section 2 lays out the time-varying DFM and categorizes the implications for forecasting when the model is subject to different types of structural instability (breaks in the factor loadings, in the factor dynamics, or in the idiosyncratic dynamics). Section 2 also reviews what little is known about factor estimation and forecasting with structural instabilities.

We then turn to an empirical examination of instability in DFMs using a new data set consisting of 144 quarterly macroeconomic time series for the United States, spanning 1959 – 2006. This data set, which is described in Section 3, improves upon earlier versions of the Stock-Watson U.S. quarterly data set by having more complete and consistent tiers of disaggregation. Motivated by the literature on the Great Moderation, we consider split-sample instability with a single break in 1984. Our forecast comparisons focus on the performance of different ways of handling this break, relative to standard full-sample factor-based forecasts (there have been numerous studies comparing full-sample factor-based forecasts to other forecasting methods and we do not repeat those exercises here, see Stock and Watson (2006) for a review). The results are summarized in Section 4. We find considerable instability in the factor loadings around the 1984 break date, but – despite this instability – principal components provides stable

estimates of the factors. In consequence, the best factor-based forecasts of individual variables use full-sample estimates of the factors but use subsample (or time-varying) estimates of the regression coefficients.

The papers most closely related to this one are Stock and Watson (1998, 2002) and Banerjee, Marcellino, and Masten (2007). Stock and Watson (2002) provide some theoretical results concerning factor estimation (but not forecasting) with time variation. Stock and Watson (1998) and Banerjee, Marcellino, and Masten (2007) provide Monte Carlo results about, respectively, estimation of factors and factor-based forecasting with instability. Banerjee, Marcellino, and Masten (2007) also report an application to data from the EU and from Slovenia, which investigates split-sample instability in the factor forecasts (but not the factor estimates themselves). Relative to these papers, the contribution here is first to lay out the implications for forecasting of different types of structural instability in DFMs, second to provide a new empirical investigation (using U.S. data) of factor-based forecasting with potential instability, and third to investigate separately the effects of structural change on the estimation of the factors and on the use of those factors for forecasting. An additional contribution is the compilation of the new quarterly data set, which is available on Watson's Web site.

2. The Time-Varying Dynamic Factor Model and Implications for Factor-Based Forecasts

This section sets out the time-varying dynamic factor model and examines the separate implications for forecasting of structural breaks in the factor loadings, in the factor dynamics, and in the idiosyncratic dynamics.

2.1 The Time-Varying Factor Model

We work with the static representation of the dynamic factor model,

$$X_t = \Lambda_t F_t + e_t, \tag{1}$$

where $X_t = (X_{1t}, ..., X_{nt})'$, F_t is a *r*-vector of static factors, Λ_t is a *n*×*r* matrix of factor loadings, and $e_t = (e_{1t}, ..., e_{nt})'$ is a *n*-vector of idiosyncratic disturbances. The difference between (1) and standard formulations of the DFM is that (1) allows for the possibility that the factor loadings can change over time.

Although parametric specifications for the factor and idiosyncratic dynamics are not needed to estimate the factors, such parametric specifications are useful when discussing forecasts using the factors. Accordingly, we specify finite-order autoregressive dynamics for the factors and idiosyncratic term:

$$F_t = \Phi_t F_{t-1} + \eta_t \tag{2}$$

$$e_{it} = a_{it}(\mathbf{L})e_{it-1} + \varepsilon_{it}, i = 1, \dots, n,$$
(3)

where η_t is a *r*-vector of factor innovations with $E(\eta_t | F_{t-1}, F_{t-2}, ..., X_{it-1}, X_{it-2}, ...) = 0$. The static factor model (1) - (3) can be derived from the dynamic factor model assuming finite lag lengths and VAR factor dynamics in the dynamic factor model, in which case F_t contains lags of the dynamic factors and Φ_t is a companion matrix so that the static factor dynamics are first order.

2.2 Time-Varying Forecast Functions with Split-Sample Time Variation

For the discussion in this subsection, suppose that $E(\varepsilon_{is}|F_t, F_{t-1},..., X_{it}, X_{it-1},...) = 0$ and $E(\eta_s|F_t, F_{t-1},..., X_{it}, X_{it-1},...) = 0$ for s > t, and that the idiosyncratic errors $\{\varepsilon_{it}\}$ are uncorrelated with the factor disturbances $\{\eta_t\}$ at all leads and lags. Then, given the data and factors through date *t*, and assuming the potentially time varying parameters are known, the *h*-step ahead conditional expectation of X_{it+h} is,

$$\begin{aligned} X_{it+h|t} &= E(X_{it+h}|F_t, F_{t-1}, \dots, X_{it}, X_{it-1}, \dots) \\ &= E(\Lambda_{t+h}F_{t+h} + e_{t+h}|F_t, F_{t-1}, \dots, X_{it}, X_{it-1}, \dots) \\ &= \beta_{it}^h \,'F_t + a_{it}^h(\mathbf{L}) \, e_{it}, \end{aligned}$$
(4)

where
$$\beta_{it}^{h'} = \Lambda_{it+h} \prod_{s=t+1}^{t+h} \Phi_s$$
 and $a_{it}^{h}(L)e_{it} = E[a_{it+h}(L)e_{t+h-1}| F_t, F_{t-1}, \dots, X_{it}, X_{it-1}, \dots] =$

 $E[e_{it+h}|e_{it}, e_{it-1},...]$, where the final equality obtains by using the factor model assumption that $\{e_{it}\}$ and $\{\eta_t\}$ are independent and by modeling expectations as linear.

Looking ahead to the empirical analysis, we consider the case of a single break at date $t = \tau$, and consider three special cases of interest, respectively corresponding to a break in Λ , Φ , and $a_{it}(L)$.

(*a*) *Forecast function with a single break in* Λ . In this case, $\Lambda_{it} = \Lambda_{i1}$, $t < \tau$, and $\Lambda_{it} = \Lambda_{i2}$, $t \ge \tau$, so (4) becomes,

$$X_{it+h|t} = \begin{cases} \beta_{i1}^{h'} F_t + a_i^h(\mathbf{L}) e_{it}, & t < \tau, \text{ where } \beta_{i1}^{h'} = \Lambda_{i1} \Phi^h \\ \beta_{i2}^{h'} F_t + a_i^h(\mathbf{L}) e_{it}, & t \ge \tau + h, \text{ where } \beta_{i2}^{h'} = \Lambda_{i2} \Phi^h \end{cases}$$
(5)

If the only break is in the factor loadings, then coefficients on F_t , but not those on e_{it} and its lags, change.

(b) Forecast function when only Φ is time-varying. In this case, $\Phi_t = \Phi_1$, $t < \tau$, and $\Phi_t = \Phi_2$, $t \ge \tau$, so (4) becomes,

$$X_{it+h|t} = \begin{cases} \beta_{i1}^{h'} F_t + a_i^h(\mathbf{L}) e_{it}, & t < \tau, \text{ where } \beta_{i1}^{h'} = \Lambda_i \Phi_1^h \\ \beta_{i2}^{h'} F_t + a_i^h(\mathbf{L}) e_{it}, & t \ge \tau + h, \text{ where } \beta_{i2}^{h'} = \Lambda_i \Phi_2^h \end{cases}$$
(6)

If the only break is in the factor dynamics, then only the coefficients on F_t change.

(c) Forecast function when only a_{it} is time-varying. In this case, $a_{it}(L) = a_{i1}(L)$, $t < \tau$, and $a_{it}(L) = a_{i2}(L)$, $t \ge \tau$, so (4) becomes,

$$X_{it+h|t} = \begin{cases} \beta_i^{h'} F_t + a_{i1}^h(\mathbf{L}) e_{it}, & t < \tau \\ \beta_i^{h'} F_t + a_{i2}^h(\mathbf{L}) e_{it}, & t \ge \tau + h \end{cases}$$
(7)

where $\beta_i^{h'} = \Lambda_i \Phi^h$. If the only break is in the idiosyncratic dynamics, then only coefficients on e_{it} and its lags change.

In certain circumstances these expressions can tell a researcher what sort of forecast instability to expect. For example, a revision of the survey used to construct a particular series X_{it} generally would result in different dynamics for the idiosyncratic term (case (c)) and possibly a change in the factor loadings (case (a)), but not a change in the factor dynamics. In general, however, the nature of the instability is not known *a-priori*. Nevertheless, by working backwards, these three cases can help identify the nature of an observed structural break. Stable factor loadings in (1), combined with a break in the coefficient on F_t in (4), point to a break in the factor dynamics. Similarly, a break in the coefficients on lagged e_{it} in (4) points to a break in the idiosyncratic dynamics.

2.3. Estimation of Static Factors in the Presence of Time Variation

The only theoretical result concerning factor estimation under model instability of which we are aware is Stock and Watson (2002), theorem 3. That result states that the factor space can be consistently estimated if there is time variation in the factor loadings, as long as that time variation is relatively small in magnitude. Monte Carlo results in Stock and Watson (1998) support this theoretical result, in fact even with quite large time variation in the factor loadings the Stock-Watson (1998) Monte Carlo experiments suggest that the factors are well estimated using principal components. That paper does not, however, consider time variation in the factor transition equation itself (Φ_t).

As the cases considered in Section 2.2 make clear, robust estimation of the factors under time variation does not imply that factor-based forecasts will be robust to time variation because of implied instability in the forecast function. This deterioration of factor-based forecasts (in contrast to the estimation of the factors themselves) is evident in Banerjee, Marcellino, and Masten's (2007) Monte Carlo results. This dichotomy – potential stability of factor estimates but instability of factor-based forecasts – is the main focus of the empirical application in Section 4.

3. The Quarterly U.S. Data Set

The empirical work employs a newly compiled data set consisting of 144 quarterly time series for the United States, spanning 1959:I – 2006:IV. The variables, sources, and transformations are listed in Appendix Table A.1. The first two quarters were used for initial values when computing first and second differences, so the data available for analysis span 1959:III – 2006:IV, for a total of T = 190 quarterly observations.

The main change in the new data set, relative to the quarterly data sets we have used in previous work, is a more complete treatment of disaggregation. The full data set contains both aggregate and subaggregate series. By construction, the idiosyncratic term of aggregate series (e.g. nonresidential investment) will be correlated with the idiosyncratic term of lower-level subaggregates (e.g. nonresidential investment – structures), and the inclusion of series related by identities (an aggregate being the sum of the subaggregates) does not provide additional information useful for factor estimation. For this reason, the factor estimates were computed using the subset of 110 series that excludes higher level aggregates related by identities to the lower level subaggregates (the series used to estimate the factors are indicated in Table A.1). This represents a departure from the approach in some previous work (e.g. Stock and Watson [2002a, 2005]) in which both aggregates and subaggregates are used to estimate the factors. The data set here includes more subaggregates than the quarterly data set in Stock and Watson (2005).

The series were transformed as needed to eliminate trends by first or second differencing (in many cases after taking logarithms); see Table A.1 for specifics.

4. Empirical Results

The empirical analysis focuses on instability around a single break in 1984:I. The reason for the 1984 break date is that 1984 (more generally, the mid-1980s) has been identified as an important break date associated with the so-called Great Moderation of output (Kim and Nelson [1999], McConnell and Perez-Quiros [2000]), and there have

been shifts in other properties of time series such as the inflation-output relation that can be dated to the mid- to late-80s (cf. Stock and Watson [2007]).

Our analysis of forecasting stability focuses on four-quarter ahead prediction. For real activity variables, the four-quarter object of interest, $X_{it+4}^{(4)}$, corresponds to growth over the next four quarters; for inflation measures, $X_{it+4}^{(4)}$ is average quarterly inflation over the next four quarters, minus inflation last quarter; and for variables entered in levels such as the capacity utilization rate, it is the value of that variable four quarters hence. Specifics are given in the appendix.

All forecasts are direct, specifically, forecasts of $X_{it+4}^{(4)}$ are obtained by regressing $X_{it+4}^{(4)}$ on variables dated *t* and earlier using the forecasting regression,

$$X_{it+4}^{(4)} = \mu_i + \beta_i' \hat{F}_t + \sum_{j=0}^{p-1} a_{ij}^4 \hat{e}_{it-j} + \text{error},$$
(8)

For comparability of results across series, p = 4 lags of \hat{e}_{it} were used for all forecasts.

4.1 The Number and Stability of the Factors

Estimates of the number of factors. Table 1 presents estimates of the number of factors, computed using criteria proposed by Bai and Ng (2002), for the full sample and the two subsamples. The results are not sharp and depend on which criterion is used. For the purposes of forecasting, 10 factors (the estimate suggested using *ICP3*) introduces a large number of parameters in the forecasting regressions so we focus on numbers of factors towards the lower end of the range in Table 1, three to five factors.

Comparison of full-sample and subsample estimated factors. Theorem 3 in Stock and Watson (2002) suggests that, despite possible time variation in the factor loadings, full- and subsample estimates of the factors could well be close, in the sense that the subsample estimates of the factor space is nearly spanned by the full-sample estimate of the factor space. This possibility is examined in Table 2, which presents the squared canonical correlations, computed over the two subsamples, between the factors estimated over the full sample and the factors estimated over the subsample. The factors

were estimated by principal components over the full sample or subsample as appropriate, always using the 110 variable dataset of subaggregates indicated in the Appendix. Canonical correlations close to one indicate that the full-sample and subsample factors span nearly the same spaces.

The results in Table 2 are consistent with there being four full sample factors and three or four factors in each subsample. If there were only two full and subsample factors (as suggested by the ICP2 results in Table 1), then one would expect the third and fourth estimated factors to have little relation to each other over the two subsamples (they would be noise), so the third canonical correlation would be low in both samples. But this is not the case, suggesting that there are at least three factors in each subsample. When four factors are estimated in both the full sample and the subsamples, the fourth canonical correlation is small in the first subsample; this is consistent with the space of three first subsample factors being spanned by the four full-sample factors, and the fourth subsample factor being noise. The moderate fourth canonical correlation in the second subsample in the case of four full and four subsample factors leads to some ambiguity, and raises the possibility that there are four factors in the second subsample, which in turn would be consistent with four factors in the full sample.

We interpret the results in Tables 1 and 2, taken together, as being consistent with there being four factors in the full sample and three factors in each subsample. The large squared canonical correlations in Table 2 for four full-sample and three subsample factors indicate that the full-sample estimated factors span the space of the three estimated factors in each subsample. Accordingly, the base case for our empirical analysis (the case used to compute all subsequent tables and figures) has four full-sample factors and three subsample factors. Still, the statistics in Table 2 alternatively could be interpreted as being consistent with other numbers of factors in the full sample and subsamples. As a robustness check, results therefore were also computed for 4 full/4 subsample, 5 full/4 subsample, and 5 full/5 subsample factors; these results are discussed briefly at the end of this section.

4.2 Stability of Factor Loadings and Forecasting Regression Coefficients

Stability of factor loadings. The stability of the factor loadings are examined in the first numeric column Table 3, which reports Chow statistics testing the hypothesis that the factor loadings are the same in the two subsamples, computed by regressing each variable onto the four full-sample estimated factors, allowing for a break in 1984:1 and using the Newey-West (1987) variance estimator (four lags). There is evidence of some instability in the factor loadings: 41% of these Chow statistics reject at the 5% significance level, and 23% reject at the 1% significance level. If one compares the results across classes of series, there are relatively fewer rejections of the stability of the factor loadings for output, employment, and inflation, and relatively more for exchange rates, term spreads, and stock returns.

Figures 1-4 focus on the stability of the estimated factors and the factor loadings for four series: real GDP growth, temporally aggregated to be the four-quarter average of the quarterly growth rates (Figure 1); the change in core PCE inflation, temporally aggregated to be the four-quarter change in inflation (Figure 2); the quarterly change in the Federal Funds rate (not temporally aggregated, Figure 3); and the term spread between the one-year and 3-month Treasury rates (not temporally aggregated, Figure 4). Part (a) of each figure presents the series, the common component computed using factors estimated from the full sample with split-sample estimates of the factor loadings (the "full-split" estimate), and the common component computed using split-sample estimates of the factors and split-sample estimates of the factor loadings ("split-split"). Part (b) presents the series, the full-split estimate of the common component, and the common component computed using factors estimated from the full sample and fullsample estimates of the factor loadings ("full-full").

In all four figures, the full-split and split-split common components (part (a)) are quite similar, consistent with the full-sample factor estimates spanning the spaces of the subsample factor estimates. There are, however, two different patterns evident in part (b) of the figures. For GDP, core PCE, and the Federal Funds rate, the full-split and full-full are similar, indicating that for those series there is little time variation in the factor loadings. This is consistent with the failure of the Chow statistic to reject the hypothesis of stable Λ 's for those three series in Table 3. In contrast, stability of the factor loadings

is rejected at the 1% significance level for the term spread, and the common components computed using the full-sample factors differ greatly depending on whether the factor loadings are estimated over the full sample or the subsample.

Stability of forecasting regressions. The remaining numeric columns of Table 3 examine the stability of the coefficients in the forecasting regression (8). Specifically, (8) was estimated by OLS using 4 lags (p = 4 in (8)), where \hat{e}_{it} in (8) was computed as the residual from the regression of X_{it} onto the full-sample factors and interactions were included to allow the coefficients to differ in the two subsamples. There is considerably more evidence for instability in the forecasting regression than in the factor loadings themselves: 84% of the Chow statistics testing the stability of all the coefficients in (8) reject at the 5% significance level, and 74% reject at the 1% significance level. If we focus on the coefficients on the factors in the forecasting regression, there is again widespread evidence of instability (72% rejections at the 5% level, 47% rejections at the 1% level). There is also evidence of instability in the idiosyncratic dynamics.

The fact that there are strikingly more rejections of stability of the coefficients on \hat{F}_t in the forecasting regressions than in the contemporaneous (factor-loading) regressions is consistent with the dynamics of the factor process changing between the two subsamples, see (6), however additional analysis is required to confirm this conjecture.

Stability of forecasting regressions by category of variable being forecasted. One possibility is that the instability evident in the forecasting equations seen in Table 3 is concentrated in a few categories of series. This possibility is explored in Table 4, which summarizes the Table 3 rejections (at the 5% significance level) by category of variable. Rejections of stability of the factor loadings are relatively less frequent for output variables, prices and wages, and money and credit variables, and are relatively more frequent for consumption, labor market, housing, and financial variables. No category, however, is immune from instability in the forecasting equations. Moreover, for all categories the instability arises more commonly from instability in the coefficients on the factors, which in turn points to instability in the dynamics of the factor process.

4.3 Subsample v. Full-Sample Forecasting Regressions

We now turn to a comparison of three different direct four-quarter ahead forecasting methods: "full-full" (that is, full-sample estimates of the factors and fullsample estimates of the forecasting regression (8), with \hat{e}_{it} the residual from the fullsample regression of X_{it} onto the four full-sample factors), "full-split" (full-sample estimates of the four full-sample factors and split-sample estimates of (8), with \hat{e}_{it} the residual from the split-sample regression of X_{it} onto the four full-sample factors), and "split-split" (split-sample estimates of the three split-sample factors and split-sample estimates of (8), with \hat{e}_{it} the residual from the split-sample regression of X_{it} onto the three split-sample factors). In each case, p = 4 in (8).

These comparisons are summarized in Table 5. Of particular interest are the relative MSEs of the three different methods, which are presented in the third and fourth column for the pre-84 sample and in the seventh and eighth column for the post-84 sample. Note that the relative MSEs are computed using the residuals from various fitted regressions, that is, these are in-sample not pseudo out-of-sample estimates; also note that the method of construction of \hat{e}_{it} and the lag specification in (8) implies that the MSE of the full-full forecast can be less than the MSE of the full-split forecast.

The relative MSEs in Table 5 are summarized in Figures 5 (pre-84 sample) and Figure 6 (post-84 sample). Part (a) of each figure is a histogram of the MSE of the full-split forecasts to the full-full forecasts. Part (b) is a histogram of the MSE of the split-split forecast to the full-split, so values exceeding 1.0 indicate that the full-split forecast outperforms the split-split forecast.

The hypothesis tests in Table 3 examined direct forecasting equations using the full-sample factors, in which the coefficients are allowed to change between the two samples; the finding from Table 3, summarized in the second column ("all coefficients") of Table 4, is that for most of the series the change in the coefficients in (8) is statistically significant. The magnitude of this improvement, measured by relative MSEs, is quantified in the "full-split to full-full" column of Table 5. As can be seen in Figures 5(a) and 6(a), allowing the forecasting coefficients to change, while using the full-sample factors, typically produces modest improvements in fit in the pre-84 sample but very substantial improvements in fit in the post-84 sample.

Given this large and statistically significant change in the forecasting coefficients using the full-sample estimates of the factors, it is natural to wonder whether one might further improve the forecasts using split-sample estimates of the factors. This possibility is examined in the "split-split to full-split" columns of Table 5 and in Figures 5(b) and 6(b). In the pre-84 sample, there is little difference on average across the series between using the full- and split-sample factors. In contrast, in the post-84 sample there are noticeable degradations on average, and substantial degradations for many series, by basing the forecasts on the split-sample factors. Strikingly, despite the evidence of some instability in the factor loadings, it is best to use all the data to estimate the factors, but to allow the coefficients of the forecasting regressions to change.

As mentioned above, there is ambiguity concerning the number of factors, and the computations underlying Tables 3 - 5 were repeated for various numbers of full-sample factors and subsample factors (specifically, 4 and 4, 5 and 4, and 5 and 5, respectively). The main findings stated above are robust to these changes in the estimated factors. The results for 4 and 4, 5 and 4, and 5 and 5 factors, like those in Table 4 for 4 and 3 factors, are also consistent with the full-sample factor estimates spanning the space of the subsample factor estimates, but the predictive regressions having coefficients which are unstable across subsamples.

6. Discussion and Conclusions

Several caveats are in order concerning the empirical results. The empirical investigation has focused on the single-break model, and multiple or continuous breaks have been ignored. The break date, 1984, has been treated as known *a-priori*, however it was chosen because of a number of interesting macroeconomic transitions that have been noticed around 1984 and thus the break date should in fact be thought of as estimated (although not on the basis of breaks in a factor model). The forecasting regressions examined here are all in-sample estimates and might not reflect out-of-sample performance. Finally, the formal theoretical justification for some of this work is limited. In particular, Stock and Watson (2002), theorem 3, only states that the space of the factors will be consistently estimated, and it does not formally justify the application of

the Bai-Ng (2002) criteria or the use of the factors as regressors (existing proofs of these have time-invariant factor loadings, e.g. Bai and Ng [2006]). These extensions of Stock and Watson (2002), theorem 3, remain a topic of ongoing research.

Despite these caveats, the results suggest three interesting conclusions. First, there is considerable evidence of instability in the factor model; the indirect evidence suggests instability in all elements (the factor loadings, the factor dynamics, and the idiosyncratic dynamics). Second, despite this instability, the factors seem to be well estimated using the full sample: the full-sample estimates of the factors span the space of the split-sample factor estimates. Third, we have the striking finding that forecasting equations using full-sample estimates of the factors and subsample estimates of the coefficients outperform forecasts using subsample estimates of both the factors and coefficients. This final finding is rather remarkable and is consistent with the theoretical results and Monte Carlo findings in Stock and Watson (1998, 2002) and Banerjeee, Marcellino, and Masten (2007). It also suggests that when factor forecasts start to break down in practical applications, attention should initially be focused on instability of the forecasting equation instead of problems with the estimates of the factors.

Data Appendix

Table A.1 lists the short name of each series, its mnemonic (the series label used in the source database), the transformation applied to the series, and a brief data description. All series are from the Global Insights Basic Economics Database, unless the source is listed (in parentheses) as TCB (The Conference Board's Indicators Database) or AC (author's calculation based on Global Insights or TCB data). The binary entry in Table A.1 the column labeled "E.F.?" indicates whether that variable was used to estimate the factors. For series available monthly, quarterly values were computed by averaging (in native units) the monthly values over the quarter. There are no missing observations.

The transformation codes in the second column of Table A.1 are defined in the following table, along with the *h*-period ahead version of the variable used in the direct forecasting regressions. In this table, Y_{it} denotes the original (native) untransformed quarterly series.

Code	Transformation (X _{it})	<i>h</i> -quarter ahead variable $X_{it}^{(h)}$
1	$X_{it} = Y_{it}$	$X_{it}^{(h)} = Y_{it+h}$
2	$X_{it} = \Delta Y_{it}$	$X_{it}^{(h)} = Y_{it+h} - Y_{it}$
3	$X_{it} = \Delta^2 Y_{it}$	$X_{it}^{(h)} = h^{-1} \sum_{j=1}^{h} \Delta Y_{i,t+h-j} - \Delta Y_{it}$
4	$X_{it} = \ln Y_{it}$	$X_{it}^{(h)} = \ln Y_{it+h}$
5	$X_{it} = \Delta \ln Y_{it}$	$X_{it}^{(h)} = \ln Y_{it+h} - \ln Y_{it}$
6	$X_{it} = \Delta^2 \ln Y_{it}$	$X_{it}^{(h)} = h^{-1} \sum_{j=1}^{h} \Delta \ln Y_{i,t+h-j} - \Delta \ln Y_{it}$

Table A.1 Data sources, transformations, and definitions

Short name	Mnemonic	Trans. Code	E.F.?	Description	
RGDP	GDP251	5	0	Real gross domestic product, quantity index (2000=100), saar	
Cons	GDP252	5	0	Real personal consumption expenditures, quantity index (2000=100), saar	
Cons-Dur	GDP253	5	1	Real personal consumption expenditures - durable goods , quantity index (2000=100), saar	
Cons-NonDur	GDP254	5	1	Real personal consumption expenditures - nondurable goods, quantity index (2000=100), saar	
Cons-Serv	GDP255	5	1	Real personal consumption expenditures - services, quantity index (2000=100), saar	
GPDInv	GDP256	5	0	Real gross private domestic investment, quantity index (2000=100), saar	
FixedInv	GDP257	5	0	Real gross private domestic investment - fixed investment, quantity index (2000=100), saar	
NonResInv	GDP258	5	0	Real gross private domestic investment - nonresidential, quantity index (2000=100), saar	
NonResInv-struct	GDP259	5	1	Real gross private domestic investment - nonresidential - structures, quantity	
NonResInv-Bequip	GDP260	5	1	Real gross private domestic investment - nonresidential - equipment & software	
Res.Inv	GDP261	5	1	Real gross private domestic investment - residential, quantity index (2000=100), saar	
Exports	GDP263	5	1	Real exports, quantity index (2000=100) , saar	
Imports	GDP264	5	1	Real imports, quantity index (2000=100), saar	
Gov	GDP265	5	0	Real government consumption expenditures & gross investment, quantity index (2000=100), saar	
Gov Fed	GDP266	5	1	Real government consumption expenditures & gross investment - federal, quantity	
Gov State/Loc	GDP267	5	1	Real government consumption expenditures & gross investment - state & local, quantity	
IP: total	IPS10	5	0	Industrial production index - total index	
IP: products	IPS11	5	0	Industrial production index - products, total	
IP: final prod	IPS299	5	0	Industrial production index - final products	
IP: cons gds	IPS12	5	0	Industrial production index - consumer goods	
IP: cons dble	IPS13	5	1	Industrial production index - durable consumer goods	
ilP:cons nondble	IPS18	5	1	Industrial production index - nondurable consumer goods	
IP:bus eqpt	IPS25	5	1	Industrial production index - business equipment	
IP: matls	IPS32	5	0	Industrial production index - materials	
IP: dble mats	IPS34	5	1	Industrial production index - durable goods materials	
IP:nondble mats IP: mfg	IPS38 IPS43	5 5	1	Industrial production index - nondurable goods materials Industrial production index - manufacturing (sic)	
IP: fuels	IPS306	5	1	Industrial production index - fuels	
NAPM prodn	PMP	1	1	Napm production index (percent)	
Capacity Util	UTL11	1	1	Capacity utilization - manufacturing (sic)	
Emp: total	CES002	5	0	Employees, nonfarm - total private	
Emp: gds prod	CES003	5	0	Employees, nonfarm - goods-producing	
Emp: mining	CES006	5	1	Employees, nonfarm - mining	
Emp: const	CES011	5	1	Employees, nonfarm - construction	
Emp: mfg	CES015	5	0	Employees, nonfarm - mfg	
Emp: dble gds	CES017	5	1	Employees, nonfarm - durable goods	
Emp: nondbles	CES033	5	1	Employees, nonfarm - nondurable goods	
Emp: services	CES046	5	1	Employees, nonfarm - service-providing	
Emp: TTU	CES048	5	1	Employees, nonfarm - trade, transport, utilities	
Emp: wholesale	CES049	5	1	Employees, nonfarm - wholesale trade	
Emp: retail	CES053	5	1	Employees, nonfarm - retail trade	
Emp: FIRE	CES088	5	1	Employees, nonfarm - financial activities	
Emp: Govt	CES140	5	1	Employees, nonfarm - government	
Help wanted indx	LHEL	2	1	Index of help-wanted advertising in newspapers (1967=100;sa)	
Help wanted/emp	LHELX	2	1	Employment: ratio; help-wanted ads:no. Unemployed clf	
Emp CPS total	LHEM	5	0	Civilian labor force: employed, total (thous.,sa)	
Emp CPS nonag	LHNAG	5	1	Civilian labor force: employed, nonagric.industries (thous.,sa)	

Emp. Hours	LBMNU	5	1	Hours of all persons: nonfarm business sec (1982=100,sa)
Avg hrs	CES151	1	1	Avg wkly hours, prod wrkrs, nonfarm - goods-producing
Overtime: mfg	CES155	2	1	Avg wkly overtime hours, prod wrkrs, nonfarm - mfg
U: all	LHUR	2	1	Unemployment rate: all workers, 16 years & over (%,sa)
U: mean duration	LHU680	2	1	Unemploy.by duration: average(mean)duration in weeks (sa)
U < 5 wks	LHU5	5	1	Unemploy.by duration: persons unempl.less than 5 wks (thous.,
		-	-	sa)
U 5-14 wks	LHU14	5	1	Unemploy.by duration: persons unempl.5 to 14 wks (thous., sa)
U 15+ wks	LHU15	5	1	Unemploy.by duration: persons unempl.15 wks + (thous., sa)
U 15-26 wks	LHU26	5	1	Unemploy.by duration: persons unempl.15 to 26 wks (thous., sa)
U 27+ wks	LHU27	5	1	Unemploy.by duration: persons unempl.27 wks + (thous, sa)
HStarts: Total	HSFR	4	0	Housing starts:nonfarm(1947-58);total farm&nonfarm(1959-)
	-		-	(thous., sa)
BuildPermits	HSBR	4	0	Housing authorized: total new priv housing units (thous., saar)
HStarts: ne	HSNE	4	1	Housing starts:northeast (thous.u.), sa
HStarts: MW	HSMW	4	1	Housing starts:midwest(thous.u.), sa
HStarts: South	HSSOU	4	1	Housing starts:south (thous.u.), sa
HStarts: West	HSWST	4	1	Housing starts:west (thous.u.), sa
PMI	PMI	1	1	Purchasing managers' index (sa)
NAPM new ordrs	PMNO	1	1	Napm new orders index (percent)
NAPM vendor del	PMDEL	1	1	Napm vendor deliveries index (percent)
NAPM Invent	PMNV	1	1	Napm inventories index (percent)
Orders (ConsGoods)	MOCMQ	5	1	New orders (net) - consumer goods & materials, 1996 dollars
(,		-		(bci)
Orders (NDCapGoods)	MSONDQ	5	1	New orders, nondefense capital goods, in 1996 dollars (bci)
PGDP	GDP272A	6	0	Gross domestic product price index
PCED	GDP273A	6	0	Personal consumption expenditures price index
CPI-All	CPIAUCSL	6	0	CPI all items (sa) fred
PCED-Core	PCEPILFE	6	0	PCE price index less food and energy (sa) (FRED)
CPI-Core	CPILFESL	6	0	CPI less food and energy (sa) (FRED)
PCED-Dur	GDP274A	6	0	Durable goods price index
PCED-Dui PCED-motorveh		6	1	
	GDP274_1			Motor vehicles and parts price index
PCED-hhequip	GDP274_2	6	1	Furniture and household equipment price index
PCED-oth dur	GDP274_3	6	1	Other price index
PCED-nondur	GDP275A	6	0	Nondurable goods price index
PCED-food	GDP275_1	6	1	Food price index
PCED-clothing	GDP275_2	6	1	Clothing and shoes price index
PCED-energy	GDP275_3	6	1	Gasoline, fuel oil, and other energy goods price index
PCED-oth nondur	GDP275_4	6	1	Other price index
PCED-services	GDP276A	6	0	Services price index
PCED-housing	GDP276_1	6	1	Housing price index
PCED-hhops	GDP276_2	6	0	Household operation price index
PCED-elect & gas	GDP276_3	6	1	Electricity and gas price index
PCED-oth hhops	GDP276_4	6	1	Other household operation price index
PCED-transport	GDP276_5	6	1	Transportation price index
PCED-medical	GDP276_6	6	1	Medical care price index
PCED-recreation	GDP276_7	6	1	Recreation price index
PCED-oth serv	GDP276_8	6	1	Other price index
PGPDI	GDP277A	6	0	Gross private domestic investment price index
PFI	GDP278A	6	0	Fixed investment price index
PFI-nonres	GDP279A	6	0	Nonresidential price index
PFI-nonres struc	GDP280A	6	1	Structures
PFI-nonres equip	GDP281A	6	1	Equipment and software price index
PFI-resdidential	GDP282A	6	1	Residential price index
PEXP	GDP284A	6	1	Exports price index
PIMP	GDP285A	6	1	Imports price index
PGOV	GDP286A	6	0	Government consumption expenditures and gross investment
	02.200.1	Ũ	Ŭ	price index
PGOV-Federal	GDP287A	6	1	Federal price index
PGOV-St & loc	GDP288A	6	1	State and local price index
Com: spot price (real)	PSCCOMR	5	1	Real spot market price index:bls & crb: all commodities
Com. spor plice (leal)		5	'	(1967=100) (psccom/PCEpilfe)
OilPrice (Real)	PW561R	5	1	Ppi crude (relative to core PCE) (pw561/PCEpilfe)
NADM com price		1	1	Nonm commodity prices index (percent)
NAPM com price	PMCP	1	1	Napm commodity prices index (percent)
NAPM com price Real AHE: goods		1 5	1 0	Real avg hrly earnings, prod wrkrs, nonfarm - goods-producing
	PMCP			

				(ces277/pi071)
Real AHE: mfg	CES278 R	5	1	Real avg hrly earnings, prod wrkrs, nonfarm - mfg (ces278/pi071)
Labor Prod	LBOUT	5	1	Output per hour all persons: business sec(1982=100,sa)
Real Comp/Hour	LBPUR7	5	1	Real compensation per hour employees: nonfarm
•				business(82=100,sa)
Unit Labor Cost	LBLCPU	5	1	Unit labor cost: nonfarm business sec (1982=100,sa)
FedFunds	FYFF	2	1	Interest rate: federal funds (effective) (% per annum,nsa)
3 mo T-bill	FYGM3	2	1	Interest rate: u.s.treasury bills,sec mkt,3-mo.(% per ann,nsa)
6 mo T-bill	FYGM6	2	0	Interest rate: u.s.treasury bills,sec mkt,6-mo.(% per ann,nsa)
1 yr T-bond	FYGT1	2	1	Interest rate: u.s.treasury const maturities,1-yr.(% per ann,nsa)
5 yr T-bond	FYGT5	2	0	Interest rate: u.s.treasury const maturities,5-yr.(% per ann,nsa)
10 yr T-bond	FYGT10	2	1	Interest rate: u.s.treasury const maturities,10-yr.(% per ann,nsa)
Aaabond	FYAAAC	2	0	Bond yield: moody's aaa corporate (% per annum)
Baa bond	FYBAAC	2	0	Bond yield: moody's baa corporate (% per annum)
fygm6-fygm3	SFYGM6	1	1	fygm6-fygm3
fygt1-fygm3	SFYGT1	1	1	fygt1-fygm3
fygt10-fygm3	SFYGT10	1	1	fygt10-fygm3
fyaaac-fygt10	SFYAAAC	1	1	fyaaac-fygt10
fybaac-fygt10	SFYBAAC	1	1	fybaac-fygt10
M1	FM1	6	1	Money stock: m1 (curr, trav.cks, dem dep, other ck'able dep)
				(bil\$,sa)
MZM	MZMSL	6	1	Mzm (sa) frb st. Louis
M2	FM2	6	1	Money stock:m2 (m1+o'nite rps, euro\$, g/p&b/d mmmfs&sav&sm
				time dep (bil\$,sa)
MB	FMFBA	6	1	Monetary base, adj for reserve requirement changes (mil\$,sa)
Reserves tot	FMRRA	6	1	Depository inst reserves:total,adj for reserve req chgs (mil\$,sa)
Reserves nonbor	FMRNBA	6	1	Depository inst reserves:nonborrowed,adj res req chgs (mil\$,sa)
Bus loans	BUSLOAN	6	1	Commercial and industrial loans at all commercial banks (FRED)
	S			billions \$ (sa)
Cons credit	CCINRV	6	1	Consumer credit outstanding – nonrevolving (g19)
Ex rate: avg	EXRUS	5	1	United States; effective exchange rate (merm) (index no.)
Ex rate: Switz	EXRSW	5	1	Foreign exchange rate: Switzerland (Swiss franc per u.s.\$)
Ex rate: Japan	EXRJAN	5	1	Foreign exchange rate: Japan (yen per u.s.\$)
Ex rate: UK	EXRUK	5	1	Foreign exchange rate: United Kingdom (cents per pound)
EX rate: Canada	EXRCAN	5	1	Foreign exchange rate: Canada (Canadian \$ per u.s.\$)
S&P 500	FSPCOM	5	1	S&P's common stock price index: composite (1941-43=10)
S&P: indust	FSPIN	5	1	S&P's common stock price index: industrials (1941-43=10)
S&P div yield	FSDXP	2	1	S&P's composite common stock: dividend yield (% per annum)
S&P PE ratio	FSPXE	2	1	S&P's composite common stock: price-earnings ratio (%, nsa)
DJIA	FSDJ	5	1	Common stock prices: Dow Jones industrial average
Consumer expect	HHSNTN	2	1	U. of Mich. index of consumer expectations (bcd-83)

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Table 1
Number of Factors Estimated Using Bai-Ng (2002) Criteria

Sample	Dates	No. Obs	Estimated Number of factors based or		rs based on:
			ICP1	ICP2	ICP3
Full	1959:III – 2006:IV	190	4	2	10
Pre-84	1959:III – 1983:IV	98	3	2	10
Post-84	1984:I – 2006:IV	92	3	2	10

Notes: All estimates use N = 110 series.

Table 2Canonical Correlations between Subsampleand Full-Sample Estimates of the Factors

Estimated	d number of	Sq	Squared canonical correlations between full and subsample factors:								
fac	ctors			Pre-84					Post-84		
Full sample	Subsample	1	2	3	4	5	1	2	3	4	5
3	3	1.00	0.99	0.03			0.99	0.91	0.84		
4	3	1.00	0.99	0.92			0.99	0.92	0.91		
4	4	1.00	0.99	0.94	0.33		1.00	0.93	0.92	0.65	
5	4	1.00	0.99	0.94	0.89		1.00	0.97	0.92	0.74	
5	5	1.00	1.00	0.94	0.90	0.49	1.00	0.97	0.93	0.79	0.11

Notes: The entries are the squared canonical correlations between the estimated factors in the indicated subsample and the factors estimated over the full sample. Factors are estimated using principal components.

Table 3. Chow Statistics Testing the Stability of the Factor Loadings and the 4-Step Ahead Forecasting Equations, 4-Factor Model

Factor loading regression: $X_{it} = \Lambda'_i \hat{F}_t + e_{it}$ Forecasting regression: $X_{i,t+4}^{(4)} = \mu_i + \beta'_i \hat{F}_t + \sum_{j=0}^3 a_{ij} \hat{e}_{it-j} + \text{error},$

where \hat{F}_{t} are the full-sample factors estimated using principal components, \hat{e}_{it} is the residual from the factor loading regression and $X_{i,t}^{(4)}$ is the 4-quarter variable to be forecast.

	Split-sample Chow statistics testing the stability of:						
Series	Factor	4-step ahe	ad forecasting	regressions:			
	loadings	All	coefficients	intercept &			
	(Λ _i)	coefficients	on <i>F</i> _t	coefficients on			
				U _{it-1}			
RGDP	5.8	36.1**	11.6*	5.8			
Cons	11.1*	50.5**	18.0**	2.5			
Cons-Dur	12.6*	60.2**	22.3**	3.4			
Cons-NonDur	9.9*	22.5**	10.2*	8.3			
Cons-Serv	5.1	69.0**	10.5*	33.8**			
GPDInv	1.6	25.1**	10.2*	7.3			
FixedInv	6.9	46.6**	28.6**	8.9			
NonResInv	5.0	27.4**	20.9**	5.2			
NonResInv-struct	5.6	17.9*	12.0*	5.4			
NonResInv-Bequip	5.9	46.0**	29.4**	12.5*			
Res.Inv	3.2	64.1**	12.5*	36.8**			
Exports	10.6*	25.8**	4.5	18.6**			
Imports	3.4	23.3**	12.2*	3.5			
Gov	7.9	7.8	3.7	3.9			
Gov Fed	12.8*	8.9	5.0	3.6			
Gov State/Loc	4.7	13.4	1.9	11.7*			
IP: total	10.7*	32.3**	12.8*	4.4			
IP: products	6.2	31.1**	11.8*	7.5			
IP: final prod	5.6	29.6**	12.3*	7.5			
IP: cons gds	11.3*	55.4**	15.3**	19.4**			
IP: cons dble	9.3	20.2*	9.0	2.1			
iIP:cons nondble	6.0	65.6**	18.8**	13.0*			
IP:bus eqpt	5.5	34.4**	21.2**	1.3			
IP: matls	9.5*	28.5**	14.4**	7.1			
IP: dble mats	8.7	28.1**	15.7**	11.5*			
IP:nondble mats	9.1	71.5**	11.0*	26.8**			
IP: mfg	9.5	33.4**	12.3*	3.7			
IP: fuels	4.1	10.3	3.4	3.7			
NAPM prodn	21.9**	36.4**	7.3	14.3*			
Capacity Util	13.0*	43.9**	26.7**	11.0			
Emp: total	25.3**	48.9**	20.9**	9.9			
Emp: gds prod	17.7**	71.8**	23.8**	21.1**			
Emp: mining	2.4	17.2*	8.7	9.4			
Emp: const	14.3**	56.9**	45.7**	16.0**			

Emp: mfg	22.4**	67.5**	21.0**	22.1**
Emp: dble gds	21.5**	75.4**	26.2**	16.8**
Emp: nondbles	7.0	79.4**	11.8*	60.1**
Emp: services	10.5*	54.0**	20.4**	15.6**
Emp: TTU	28.0**	80.1**	34.8**	24.6**
Emp: wholesale	29.2**	76.9**	35.1**	22.4**
Emp: retail	11.8*	170.6**	48.1**	58.5**
Emp: FIRE	16.2**	99.5**	31.9**	38.9**
Emp: Govt	31.0**	30.3**	11.1*	23.0**
Help wanted indx	14.3**	55.7**	7.5	26.8**
		24.8**		
Help wanted/emp	<u> </u>	24.0	7.4 14.6**	<u> </u>
Emp CPS total				
Emp CPS nonag	6.4	34.2**	11.2*	17.8**
Emp. Hours	28.1**	69.8**	31.9**	9.4
Avg hrs	6.6	89.4**	9.1	70.0**
Overtime: mfg	2.1	20.9*	3.0	8.3
U: all	10.6*	26.3**	22.5**	2.5
U: mean duration	5.6	55.7**	15.2**	27.4**
U < 5 wks	16.1**	13.7	10.7*	2.7
U 5-14 wks	5.5	17.5*	15.6**	0.9
U 15+ wks	1.5	27.2**	18.1**	11.3*
U 15-26 wks	3.1	27.5**	14.9**	12.0*
U 27+ wks	0.4	32.1**	15.8**	18.1**
HStarts: Total	11.2*	35.9**	8.7	14.2*
BuildPermits	9.9*	25.0**	9.8*	6.0
HStarts: ne	1.7	42.2**	9.3	25.7**
HStarts: MW	23.4**	20.2*	10.5*	5.2
HStarts: South	18.1**	29.6**	19.6**	8.0
HStarts: West	7.7	26.5**	18.0**	4.1
PMI	24.9**	31.6**	8.9	13.6*
NAPM new ordrs	40.7**	28.3**	4.8	16.1**
NAPM vendor del	14.0**	17.5*	12.1*	6.0
NAPM Invent	18.1**	75.8**	16.8**	50.8**
Orders (ConsGoods)	11.7*	38.9**	14.0**	12.8*
Orders (NDCapGoods)	6.1	33.3**	23.5**	6.2
PGDP	9.8*	32.4**	26.5**	1.0
PCED	2.0	23.8**	18.8**	3.6
CPI-All	7.5	32.9**	22.0**	5.4
PCED-Core	6.7	29.5**	24.0**	5.6
CPI-Core	19.3**	14.1	9.9*	5.4
PCED-Dur	2.2	17.2*	11.6*	2.8
PCED-motorveh	2.5	9.2	6.7	3.3
PCED-hhequip	9.0	71.9**	61.2**	14.2*
PCED-oth dur	3.2	25.1**	13.4**	16.3**
PCED-nondur	2.8	23.2**	10.6*	2.9
PCED-food	5.3	34.6**	22.7**	5.9
PCED-clothing	2.1	10.1	4.4	3.9
PCED-energy	7.7	44.7**	26.5**	4.0
PCED-oth nondur	5.9	17.8*	2.2	14.5*
PCED-services	4.6	57.7**	45.8**	4.3
PCED-housing	2.6	5.7	4.1	2.7
PCED-hhops	4.5	13.0	8.7	4.1
PCED-elect & gas	4.8	9.7	3.9	3.1
	1.0	5.7	0.0	0.1

PCED-oth hhops	2.2	12.3	3.1	4.9
PCED-transport	9.5	76.2**	16.4**	44.9**
PCED-medical	24.2**	34.3**	11.8*	12.7*
PCED-recreation	5.8	14.5	8.0	8.0
PCED-oth serv	8.6	25.5**	9.3	7.3
PGPDI	8.4	21.4*	16.6**	2.8
PFI	5.9	27.3**	15.8**	7.4
PFI-nonres	4.5	32.1**	12.9*	20.2**
PFI-nonres struc	6.2	14.2	6.1	9.0
PFI-nonres equip	3.6	13.9	10.8*	2.3
PFI-resdidential	4.7	59.5**	21.3**	10.5
PEXP	5.1	23.8**	11.4*	14.3*
PIMP	4.3	27.1**	16.2**	1.3
PGOV	3.0	22.6**	16.8**	7.0
PGOV-Federal	1.6	23.6**		5.5
	3.3	28.6**	6.1 24.0**	4.6
PGOV-St & loc				
Com: spot price (real)	8.1	30.1**	15.3**	10.0
OilPrice (Real)	26.9**	24.6**	12.9*	11.5*
NAPM com price	8.7	104.0**	22.2**	62.0**
Real AHE: goods	5.0	58.0**	11.9*	36.1**
Real AHE: const	13.3**	38.5**	22.6**	6.2
Real AHE: mfg	8.0	54.8**	9.4	27.1**
Labor Prod	10.6*	8.4	5.0	1.7
Real Comp/Hour	12.5*	8.7	5.1	4.0
Unit Labor Cost	18.3**	45.4**	9.0	37.7**
FedFunds	8.6	48.5**	33.1**	12.6*
3 mo T-bill	4.7	43.7**	32.4**	11.5*
6 mo T-bill	15.4**	32.6**	16.8**	12.7*
1 yr T-bond	14.8**	22.8**	12.0*	12.2*
5 yr T-bond	8.2	9.9	1.4	7.5
10 yr T-bond	6.1	13.4	1.1	7.2
Aaabond	9.6*	14.4	4.3	6.3
Baa bond	11.3*	17.6*	7.7	5.1
fygm6-fygm3	22.7**	34.6**	4.2	24.5**
fygt1-fygm3	23.2**	55.7**	25.7**	14.1*
fygt10-fygm3	17.4**	26.4**	9.7*	7.5
fyaaac-fygt10	4.9	60.4**	11.7*	35.5**
fybaac-fygt10	15.2**	57.5**	33.5**	11.6*
M1	2.5	11.6	2.8	4.6
MZM	5.3	13.2	7.1	4.0
M2	13.0*	55.5**	40.3**	6.7
MB	8.1	34.4**	12.2*	21.5**
Reserves tot	4.6	49.2**	9.2	22.4**
Reserves nonbor	8.7	16.1	12.1*	5.7
Bus loans	3.2	38.0**	15.3**	10.7
Cons credit	3.3	20.5*	15.9**	2.6
Ex rate: avg	26.8**	21.0*	11.0*	4.1
Ex rate: Switz	9.6*	17.0*	8.0	9.7
Ex rate: Japan	6.4	26.0**	9.6*	10.0
Ex rate: UK	6.4	43.4**	13.5**	10.6
EX rate: Canada	6.4	26.5**	19.3**	6.2
S&P 500	11.0*	22.2**	12.4*	6.1
			13.3*	5.7

S&P div yield	11.3*	21.8**	15.2**	5.5
S&P PE ratio	18.6**	56.6**	37.1**	7.3
DJIA	6.8	33.0**	14.3**	15.4**
Consumer expect	23.5**	38.0**	18.4**	10.2

Notes: Entries are chi-squared Chow statistics computed using Newey-West (1987) standard errors with 4 lags (column 1) and 5 lags (columns 2-4). Asterisks indicate that the Chow statistics exceed standard *5% and **1% critical values.

Table 4.

Summary of Chow tests by category of variable: Fraction rejections of variables
within category at the 5% significance level

Category	Number	Split-sample Chow statistics testing the stability of:						
	of	Factor	4-step ahead forecasting regressions:					
	series	loadings	All	coefficients	intercept &			
		(Λ _i)	coefficients	on <i>F</i> _t	coefficients			
					on u _{it-1}			
Output	14	0.29	0.93	0.79	0.36			
Consumption	4	0.75	1.00	1.00	0.25			
Labor market	27	0.59	0.96	0.81	0.74			
Housing	7	0.57	1.00	0.71	0.43			
Investment,	11	0.45	1.00	0.82	0.45			
inventories, & orders								
Prices & wages	42	0.17	0.74	0.67	0.29			
Financial variables	23	0.61	0.87	0.74	0.39			
Money & credit	8	0.13	0.63	0.63	0.25			
Other	8	0.63	0.50	0.38	0.25			
All	144	0.41	0.84	0.72	0.41			

Table 5.

Root Mean Square Errors (RMSEs) and Relative MSEs of 4-step ahead Forecasting Regressions: 4 Full-Sample Factors, 3 Subsample Factors

The forecasting regressions (specification (8)) are estimated using:

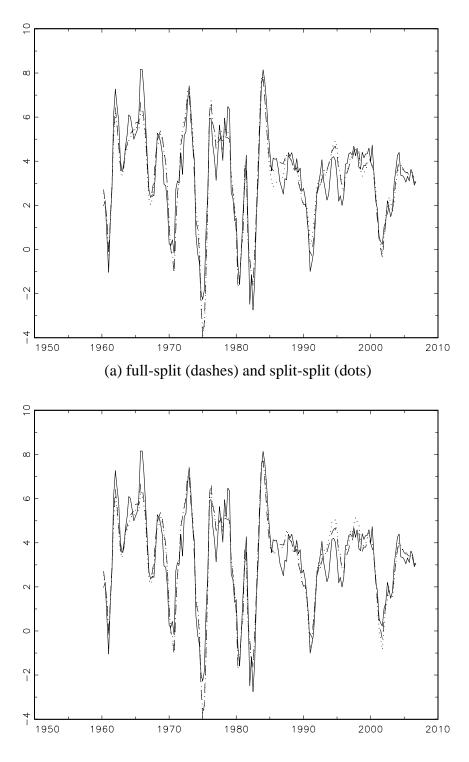
- (a) full-sample factor estimates and full-sample coefficients ("full-full")
- (b) full-sample factor estimates and split-sample coefficients ("full-split")
- (c) split-sample factor estimates and full-sample coefficients ("split-split")

	Pre-84 Sample				Post-84 Sample				
Series (X _{it})	eries (X _{it}) Std MSE ratio		ratio	Std		MSE ratio			
	dev	RMS	full-	split-	dev of	RMSE,	full-	split-	
	of	Ε,	split	split	$X_{it}^{(4)}$	full-full	split	split	
	$X_{it}^{(4)}$	full-	to full-	to full-	_{it}		to full-	to full-	
		full	full	split			full	split	
RGDP	2.73	2.13	0.94	0.99	1.29	1.23	0.69	1.17	
Cons	2.16	1.80	0.95	0.99	1.11	1.08	0.71	1.16	
Cons-Dur	7.59	5.71	0.94	0.99	4.42	4.47	0.83	1.05	
Cons-NonDur	2.01	1.75	0.88	1.10	1.18	1.18	0.77	1.14	
Cons-Serv	1.26	1.17	0.90	0.98	0.86	0.84	0.54	1.29	
GPDInv	11.97	8.28	0.90	1.01	6.72	6.27	0.80	1.07	
FixedInv	7.85	5.73	0.89	1.00	5.10	4.60	0.69	1.04	
NonResInv	7.47	5.43	0.87	1.03	6.14	4.87	0.76	0.99	
NonResInv-struct	7.65	6.62	0.87	1.00	7.71	6.17	0.80	1.01	
NonResInv-Bequip	8.33	5.80	0.86	1.04	6.09	5.07	0.72	1.01	
Res.Inv	16.88	12.11	0.95	1.00	7.25	7.20	0.62	1.18	
Exports	6.76	5.34	0.92	0.98	5.27	5.09	0.88	1.01	
Imports	8.63	5.81	0.96	1.03	4.56	3.97	0.86	1.04	
Gov	2.85	2.48	1.00	1.00	1.77	1.49	0.93	0.99	
Gov Fed	5.07	4.34	1.00	1.00	3.54	2.87	0.90	0.94	
Gov State/Loc	2.51	2.08	0.99	1.00	1.61	1.32	0.82	1.05	
IP: total	5.37	3.68	0.93	1.00	2.80	2.56	0.76	1.05	
IP: products	4.58	3.25	0.92	0.99	2.46	2.23	0.74	1.09	
IP: final prod	4.50	3.26	0.91	1.00	2.42	2.25	0.73	1.06	
IP: cons gds	4.05	2.62	0.96	1.02	1.70	1.88	0.56	1.18	
IP: cons dble	9.46	6.63	0.97	0.99	4.80	4.49	0.85	1.08	
iIP:cons nondble	2.38	2.01	0.88	1.12	1.40	1.62	0.51	1.20	
IP:bus eqpt	8.29	5.34	0.89	1.03	5.88	4.84	0.86	1.01	
IP: matls	6.48	4.41	0.93	0.98	3.42	3.25	0.75	0.99	
IP: dble mats	9.70	6.43	0.93	1.01	5.52	5.09	0.73	1.03	
IP:nondble mats	5.91	4.48	0.85	1.02	2.91	3.19	0.60	1.13	
IP: mfg	6.00	4.08	0.93	0.99	3.18	2.84	0.78	1.06	
IP: fuels	5.19	5.05	0.96	1.00	3.52	3.41	0.81	1.06	
NAPM prodn	8.00	6.97	0.96	0.98	5.56	5.27	0.80	1.20	
Capacity Util	5.35	3.01	0.90	1.00	3.19	2.15	0.73	1.12	
Emp: total	2.36	1.61	0.89	0.96	1.53	1.00	0.61	1.15	
Emp: gds prod	4.20	2.78	0.90	0.97	2.44	1.79	0.58	1.13	
Emp: mining	6.69	6.33	0.93	1.01	6.41	5.50	0.83	1.03	
Emp: const	5.45	3.99	0.92	0.98	3.89	2.87	0.70	1.09	
Emp: mfg	4.26	2.97	0.86	0.98	2.48	2.03	0.49	1.11	
Emp: dble gds	5.48	3.75	0.87	0.99	3.11	2.42	0.56	1.07	
Emp: nondbles	2.57	2.03	0.74	1.04	1.90	1.47	0.53	1.08	
Emp: services	1.33	0.87	0.87	0.98	1.13	0.68	0.70	1.15	

Emp: TTU	1.78	1.26	0.81	0.99	1.59	1.06	0.62	1.19
Emp: wholesale	1.88	1.44	0.71	1.04	1.86	1.29	0.02	1.10
Emp: retail	1.74	1.28	0.79	1.04	1.64	1.21	0.58	1.19
Emp: FIRE	1.29	0.88	0.85	1.01	1.63	1.19	0.75	1.12
Emp: Govt	1.93	1.25	0.94	1.01	0.80	0.85	0.65	1.00
Help wanted indx	3.46	2.68	0.84	1.02	2.44	1.87	0.81	1.13
Help wanted/emp	0.09	0.07	0.97	1.01	0.04	0.04	0.71	1.07
Emp CPS total	1.55	1.15	0.86	0.99	0.98	0.78	0.65	1.38
Emp CPS nonag	1.58	1.15	0.84	0.98	1.03	0.78	0.64	1.38
Emp. Hours	2.70	1.92	0.85	0.98	1.98	1.61	0.68	1.08
Avg hrs	0.50	0.35	0.85	0.98	0.42	0.31	0.89	0.99
Overtime: mfg	0.30	0.08	0.98	1.00	0.42	0.07	0.89	1.06
U: all	0.12	0.08	0.95	1.00	0.08	0.07	0.91	1.23
	0.55		0.95					
U: mean duration U < 5 wks		0.29		1.03	0.43	0.25	0.68	1.17
	9.85	8.13	0.93	1.02	6.50	6.14	0.85	1.10
U 5-14 wks	21.00	15.44	0.96	1.01	11.52	9.60	0.76	1.24
U 15+ wks	38.50	23.62	0.93	1.00	22.77	15.14	0.65	1.18
U 15-26 wks	34.09	22.62	0.94	1.00	19.93	15.23	0.68	1.24
U 27+ wks	46.91	27.03	0.95	1.02	27.70	16.88	0.67	1.23
HStarts: Total	0.23	0.19	0.94	1.01	0.18	0.12	0.78	0.99
BuildPermits	0.26	0.21	0.98	0.98	0.21	0.13	0.77	0.98
HStarts: ne	0.30	0.21	0.96	0.97	0.27	0.15	0.79	1.10
HStarts: MW	0.32	0.25	1.00	1.00	0.14	0.11	0.98	1.08
HStarts: South	0.26	0.19	0.97	0.92	0.23	0.13	0.76	1.03
HStarts: West	0.33	0.24	0.99	1.00	0.20	0.14	0.84	1.03
PMI	7.82	6.70	0.93	0.94	4.66	4.55	0.73	1.22
NAPM new ordrs	8.58	7.38	0.98	0.99	5.85	5.43	0.80	1.23
NAPM vendor del	13.51	11.12	0.95	0.97	4.66	5.17	0.56	1.18
NAPM Invent	7.68	6.39	0.84	0.90	3.15	3.59	0.42	1.22
Orders (ConsGoods)	8.51	6.34	0.87	0.96	3.49	3.61	0.68	1.06
Orders (NDCapGoods)	15.02	10.98	0.89	1.01	9.89	8.66	0.78	1.00
PGDP	1.43	0.99	0.96	0.99	0.73	0.59	0.63	1.13
PCED	1.49	1.17	0.97	0.98	0.99	0.80	0.69	1.10
CPI-All	1.98	1.33	0.95	1.00	1.39	1.14	0.70	1.02
PCED-Core	1.24	0.98	0.98	1.01	0.60	0.49	0.60	1.16
CPI-Core	1.99	1.74	0.98	1.04	0.55	0.56	0.54	1.07
PCED-Dur	2.50	1.81	0.95	1.05	1.33	1.26	0.63	1.19
PCED-motorveh	4.17	2.86	0.98	1.02	2.30	1.89	0.83	1.04
PCED-hhequip	1.92	1.44	0.91	1.08	1.82	1.47	0.59	1.15
PCED-oth dur	2.87	2.36	0.96	1.02	2.00	1.34	0.72	1.28
PCED-nondur	2.59	1.99	0.95	0.95	2.95	1.99	0.90	1.03
PCED-food	3.28	2.33	1.00	0.98	1.24	0.99	0.75	1.16
PCED-clothing	2.14	1.58	0.95	1.07	3.03	1.78	0.87	1.08
PCED-energy	14.29	11.06	0.83	0.99	27.93	18.87	1.01	0.93
PCED-oth nondur	2.49	1.91	0.91	1.04	1.59	1.19	0.75	1.09
PCED-services	1.21	0.91	0.98	0.97	0.82	0.55	0.76	1.00
PCED-housing	1.22	0.97	0.98	0.98	0.81	0.63	0.90	1.04
PCED-hhops	2.40	1.82	0.92	0.98	3.50	2.31	0.94	1.07
PCED-elect & gas	3.78	2.90	0.70	1.02	7.30	5.90	0.92	1.01
PCED-oth hhops	2.74	2.23	0.97	1.01	1.72	1.19	0.78	1.16
PCED-transport	6.80	5.04	0.57	1.07	6.60	7.15	0.71	0.99
PCED-medical	1.80	1.42	0.93	1.00	0.94	0.97	0.71	1.01
PCED-recreation	1.72	1.13	1.03	0.97	1.10	0.77	0.87	1.08

PCED-oth serv	2.59	2.13	0.96	1.00	2.71	1.97	0.76	0.84
PGPDI	2.63	1.72	0.95	1.08	1.25	1.19	0.54	1.13
PFI	2.66	1.75	0.94	1.06	1.29	1.20	0.55	1.11
PFI-nonres	2.60	1.89	0.91	1.07	1.32	1.23	0.59	1.08
PFI-nonres struc	3.68	2.90	0.96	1.01	2.12	1.81	0.73	1.08
PFI-nonres equip	2.74	1.92	0.90	1.09	1.62	1.46	0.66	1.06
PFI-resdidential	4.53	4.08	0.98	1.00	2.21	1.94	0.43	1.00
PEXP	5.17	3.93	0.97	0.95	2.38	2.19	0.69	1.11
PIMP	8.49	7.55	0.95	0.96	6.58	4.79	0.83	1.00
PGOV	2.29	1.34	0.89	1.00	1.62	1.11	0.71	1.02
PGOV-Federal	3.89	1.86	0.95	1.00	2.72	1.25	0.87	0.99
PGOV-St & loc	1.94	1.40	0.89	0.97	1.55	1.27	0.68	1.06
Com: spot price (real)	12.85	9.93	0.88	1.06	9.21	8.58	0.00	1.06
OilPrice (Real)	11.51	11.16	0.72	1.00	24.19	21.91	0.82	1.00
NAPM com price	12.95	11.10	0.72	0.94	13.22	13.50	0.66	1.14
Real AHE: goods	1.49	1.37	0.80	1.06	1.16	0.86	0.00	1.09
Real AHE: const	2.60	1.93	0.91	1.00	1.43	1.20	0.80	0.97
Real AHE: mfg	1.40	1.36	0.98	1.02	1.43	0.92	0.80	1.09
Labor Prod	1.95	1.76	0.87	1.04	1.28	1.17	0.72	1.09
Real Comp/Hour	1.95	1.11	0.95	1.03	1.58	1.17	0.84	1.00
Unit Labor Cost	3.74	2.43	1.01	0.94	1.38	1.53	0.50	1.01
FedFunds	0.63	0.44	0.89	0.94	0.38	0.32	0.66	1.03
3 mo T-bill	0.03	0.44	0.89	0.97	0.35	0.32	0.00	1.03
6 mo T-bill	0.45	0.33	0.87	1.06	0.35	0.31	0.71	1.05
1 yr T-bond	0.45	0.37	0.88	1.08	0.35	0.32	0.72	1.00
5 yr T-bond	0.40	0.39	0.89	1.08	0.30	0.30	0.79	0.94
10 yr T-bond	0.34	0.31	0.93	1.04	0.30	0.30	0.89	0.94
Aaabond	0.29	0.28	0.92	1.02	0.27	0.27	0.86	0.92
Baa bond	0.20	0.24	0.93	1.03	0.21	0.22	0.86	0.92
	0.30	0.20	0.92	1.03	0.21	0.21	0.88	1.12
fygm6-fygm3	0.22		0.95		0.14		0.73	
fygt1-fygm3	1.20	0.40 0.93	0.85	1.08 1.01	1.12	0.33 0.83	0.70	1.09
fygt10-fygm3 fyaaac-fygt10	0.34	0.93	0.95	1.01	0.40	0.83	0.70	0.99 1.02
	0.34	0.30	0.81		0.40		0.84	1.02
fybaac-fygt10 M1	3.16	2.08	0.89	0.99 1.01	4.40	0.41 3.77	0.84	
MZM	5.97	5.29	0.87	0.96	4.40 5.08	4.61	0.94	0.84 0.81
M2	3.09	2.23	0.90	1.03	2.49	2.23	0.81	0.84
MB	1.82	1.41	0.81	0.98	2.49	2.23	0.96	0.84
Reserves tot	5.25	4.02	0.60	0.98	8.64	7.43	0.90	0.97
	12.74	12.73	0.00	1.08	14.49	13.04	0.84	1.03
Reserves nonbor	6.71	4.90	0.91	1.08	4.91	4.07	0.70	
Bus loans Cons credit	4.23	4.90	0.91	1.03	3.48	4.07 3.37	0.79	1.08
	4.23	3.07 4.51	0.87	0.97	3.48 7.62	<u>3.37</u> 6.97	0.84	1.01
Ex rate: avg	9.70		0.86		12.49		0.90	1.14
Ex rate: Switz	8.71	9.13 7.93	0.90	1.05 1.13	12.49	11.69 11.72	0.89	1.05 1.06
Ex rate: Japan Ex rate: UK	9.05	8.29	0.87	1.13	9.12	8.99	0.92	1.06
EX rate: Canada	3.37	3.69	0.78	1.01	9.12 5.58	<u> </u>	0.77	0.96
S&P 500	14.28	12.57	0.75	1.04	14.21	4.55	0.93	1.00
S&P 500 S&P: indust	14.20	12.57	0.79	1.05	14.21	14.72	0.74	1.00
S&P div yield	0.17	0.12	0.80	1.12			0.76	
		0.12			0.09 1.27	0.10		0.99
S&P PE ratio	0.68		0.69	1.12		1.07	0.79	1.01
DJIA Consumer expect	14.09	11.83	0.78	1.03	13.06	14.01	0.67	1.00
Consumer expect	2.92	2.12	0.83	1.01	2.46	2.52	0.69	1.01

Figure 1. 4-Quarter real GDP growth (solid line) and three estimates of its common component: split sample factors, split sample factor loadings (split-split); full sample factors, split sample factor loadings (full-split); and full sample factors, full sample factor loadings (full-full).



(b) full-split (dashes) and full-full (dots)

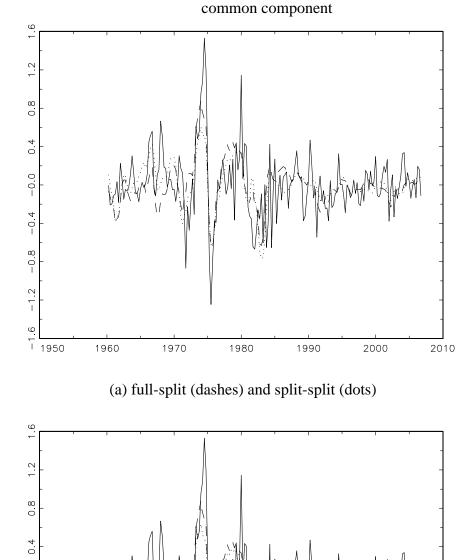


Figure 2. Four-quarter change in core PCE inflation (solid line) and three estimates of its common component

(b) full-split (dashes) and full-full (dots)

1980

1990

2000

2010

0.0-

-0.4

-0.8

-1.2

<u>ب</u> 1950

1960

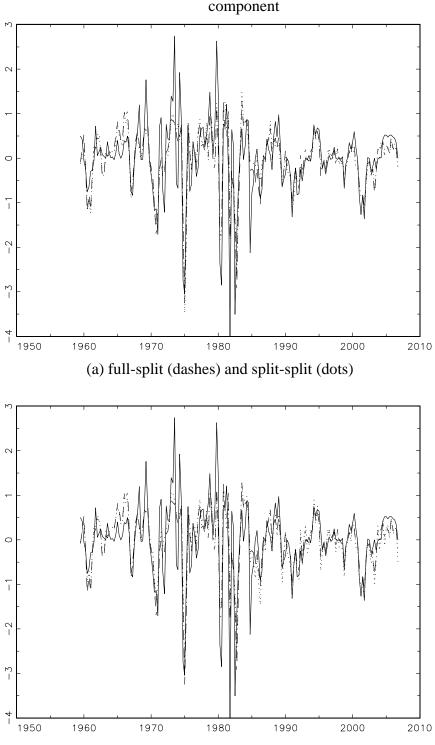


Figure 3. The Federal Funds rate (solid line) and three estimates of its common component

(b) full-split (dashes) and full-full (dots)

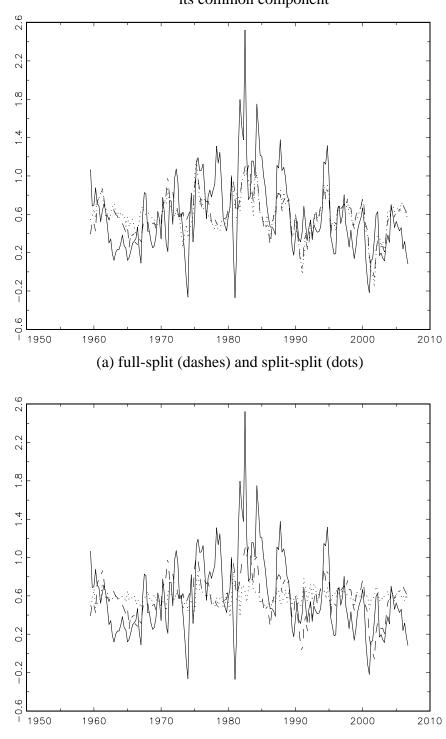
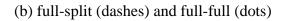


Figure 4 The one-year/3-month Treasury term spread (solid line) and three estimates of its common component



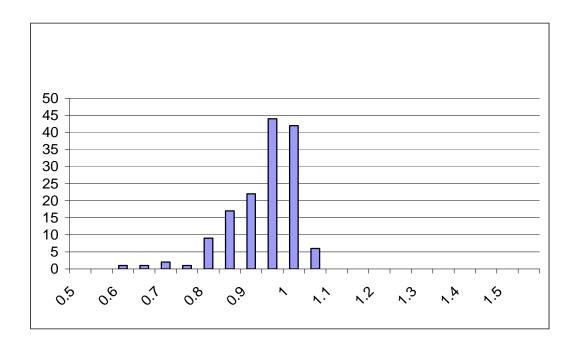


Figure 5(a). Histogram of relative MSEs of full-split forecasts to full-full forecasts, pre-1984 sample (mean = .91, median = .92)

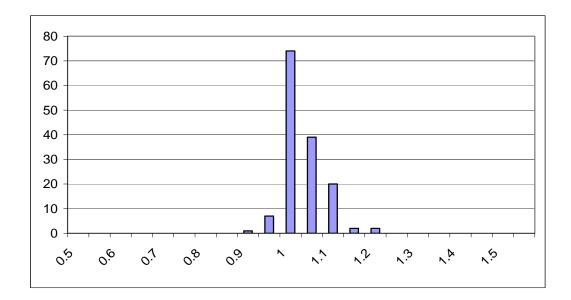


Figure 5(b). Histogram of relative MSEs of split-split forecasts to full-split forecasts, pre-1984 sample (mean = 1.01, median = 1.00)

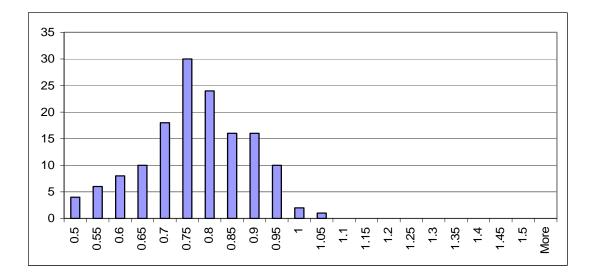


Figure 6(a). Histogram of relative MSEs of full-split forecasts to full-full forecasts, post-1984 sample (mean = .75, median = .75)

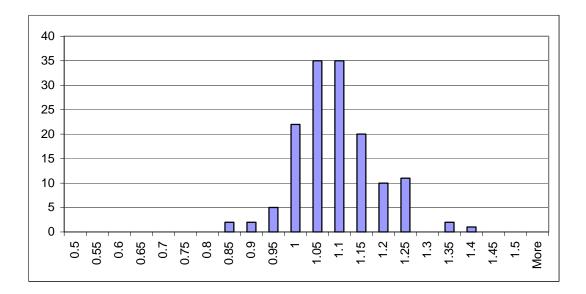


Figure 6(b). Histogram of relative MSEs of split-split forecasts to full-split forecasts, post-1984 sample (mean = 1.07, median = 1.06)