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BUILDING BLOCKS FOR YOUNG CHILDREN'S MATHEMATICAL DEVELOPMENT*

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ABSTRACT

This article describes the design principles behind a set of research-based software microworlds included in the *Building Blocks* program, a PreK to grade 2 software-based mathematics curriculum. *Building Blocks*' approach is finding the mathematics in, and developing mathematics from, children's activity. The materials are designed to help children extend and mathematize their everyday activities, from building blocks to art to songs and stories to puzzles. The 9-step design process model that defines what we mean by "research-based" is described and illustrated.

Building Blocks is a PreK to grade 2 software-based mathematics curriculum development project, designed to comprehensively address the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). This article describes the design principles behind a set of research-based

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software microworlds included in the *Building Blocks* program. We begin with a justification for the project's software, including a discussion of what it means to conduct "research-based" curriculum development. We then examine the design principles of the *Building Blocks* project and provide examples.

RATIONALE FOR THE *BUILDING BLOCKS PROJECT*

First, there is a need to examine whether there is a need for additional software in early childhood mathematics. Hundreds of products are now available for young children that include mathematics activities in one form or another. However, most of these products fall into one of two categories. First, "edutainment" has attractive multimedia features but limited mathematics content and pedagogy (even the drill in these packages is badly designed, not using the computer's management capabilities). Second, there are discovery-oriented environments that appear interesting, but are usually explored only on the surface level by young children. In both cases, there is little learning, by children or by those in the education field (Clements & Battista, 2000; Clements & Sarama, 2002).

In either case, theoretical and empirical support for the use of the software to support learning is usually lacking. And this speaks to one of the main points of this article: The necessity of creating and, further, insisting upon, research-based curricula and educational software. We contend that research-based curriculum development efforts can contribute to (a) more effective curriculum materials because the research reveals critical issues for instruction, (b) better understanding of students' mathematical thinking, and (c) research-based change in mathematics curriculum (Clements, Battista, Sarama, & Swaminathan, 1997; Schoenfeld, 1999). Indeed, we believe that education will not improve substantially without a system-wide commitment to research-based curriculum and software development.

Let us be specific here: Many software developers do claim a research basis for their materials, but these claims are often vacuous. We have identified nine possibilities (Clements, 2002; Sarama & Clements, 2001).

1. Broad philosophies, theories, and empirical results on learning and teaching are considered when creating curriculum.
2. Empirical findings on making activities educationally effective—motivating and efficacious—serve as general guidelines for the generation of activities.
3. Research is used to identify mathematics that is developmentally appropriate and interesting to students in the target population.
4. Activities are structured to be consistent with empirically-based learning models of children's thinking and learning.

5. Sets of activities are sequenced according to learning trajectories through the concepts and skills that constitute a domain of mathematics.
6. Activities or activity sets are extensively field-tested from their first inception and early intensive interpretive work, to classroom-based studies, and are revised substantially after each iteration.
7. Summative evaluation studies are conducted, including issues of scalability.
8. Each phase of the development process is documented, reflected upon, analyzed, and reported in the scientific literature.

Another mode that is spurious, but probably frequent in practice, should be mentioned for completeness.

9. Following the creation of a curriculum, research results that are ostensibly consistent with it are cited post hoc.

Given this variety of possibilities, claims that a curriculum is based on research should be questioned to reveal the exact nature between the two. Further, to realize the full potential of "research and development" for gaining knowledge, we need to add another process. We contend that research has played a minimum role in the development of most software packages. In the majority of cases, testing the software with target users is rare (which may account for the generally low quality of the software). Often, there is only minimal formative research, such as a polling of easily accessible peers, rather than any systematic testing with an appropriate target audience. "Beta" testing is done sometimes, but late enough in the process that changes are minimal, given the time and resources dedicated to the project already and the limited budget and pressing deadlines that remain (Char, 1989; Clements & Battista, 2000). Such testing is more summative than formative (Schauble, 1990).

Even when conducted, most summative evaluations are limited in scope. The majority of studies have used traditional quantitative designs in which the "computer" was the "treatment." The general conclusion drawn was that such treatments lead to moderate but statistically significant learning gains, especially in mathematics (Becker, 1992; Clements & Nastasi, 1992; Kelman, 1990; Roblyer, Castine, & King, 1988). However, this conclusion must be tempered with the realization that most of the software used presented drill and practice exercises. Therefore, the potential of software based on different approaches to learning, such as mathematical microworlds (as well as the appropriateness of the methodology for evaluating such different types of software), has not been addressed adequately.

In contrast, *Building Blocks* is structured on empirically-based learning trajectories through the big ideas and skill areas of mathematics (Clements & Battista, 1992; Fuson, 1997). It applies research on making computer software for young children motivating and educationally effective (Clements, Nastasi, & Swaminathan, 1993; Clements & Swaminathan, 1995). It includes mathematics that research identifies as developmentally appropriate for and interesting to young children. Finally, it is based on a design process model that includes specification of mathematical ideas (computer objects) and processes/skills (computer tools) and extensive field-testing from the first inception through to large summative evaluation studies (Clements & Battista, 2000). The next section describes the phases of this design process model.

A MODEL FOR THE DEVELOPMENT OF RESEARCH-BASED MICROWORLDS

In our model, research is conducted at multiple levels, with the goal of making the research relevant to educators in many positions. Feedback from the field continually results in further refinement of the design of the software and activities, which then results in further testing. In this way, we continually loop through the earlier phases of the model.

Phase 1: Draft the Initial Goals

The first phase begins with the identification of a significant domain of mathematics. The learning of the domain should make a substantive contribution to students' mathematical development. Learning about students' mathematical activity in the domain should make a similar contribution to research and theory.

One of the reasons underlying the name we gave to our project was our desire that the materials emphasize the development of basic *mathematical building blocks*—ways of knowing the world mathematically—organized into two areas: (a) spatial and geometric competencies and concepts and (b) numeric and quantitative concepts, based on the considerable research in that domain. Research shows that young children are endowed with intuitive and informal capabilities in both these areas (Bransford, Brown, & Cocking, 1999; Clements, 1999a). For example, research shows that preschoolers know a considerable amount about shapes (Clements, Swaminathan, Hannibal, & Sarama, 1999; Lehrer, Jenkins, & Osana, 1998), and they can do more than we assume, especially working with computers (Sarama, Clements, & Vukelic, 1996). In the broad area of geometry and space, they can do the following: recognize, name, build, draw, describe, compare, and sort two- and three-dimensional shapes, investigate putting shapes together and taking them apart, recognize and use slides and turns, describe spatial locations such as "above" and "behind," and describe, and use ideas of direction and distance in getting around in their environment (Clements, 1999a). In the area of number, preschoolers can learn to count with understanding (Fuson, 1988; Gelman, 1994), recognize "how many" in small sets of objects (Clements, 1999b; Reich, Subrahmanyam, & Gelman, 1999), compare numbers (Griffin, Case, & Capodilupo, 1995), and learn simple ideas of addition and subtraction (Aubrey, 1997; Clements, 1984; Siegler, 1996). They can count higher and generally participate in a much more exciting and varied mathematics than usually considered (Ginsburg, Inoue, & Seo, 1999; Trafton, 1997). Challenging number activities don't just develop children's number sense; they can also develop children's competencies in such logical processes as sorting and ordering (Clements, 1984). Three mathematical themes are woven through both of these main areas: (a) patterns, (b) data, and (c) sorting and sequencing.

We focus on a single area for this article and a single related microworld from the many included in *Building Blocks.* Early in the project, we determined that a basic, often neglected, area of children's mathematics was the composition and decomposition of two-dimensional geometric figures (other domains in geometry include shapes and their properties, transformations/congruence, and measurement). The geometric composition domain was determined to be significant for students in two ways. First, it is a basic geometric competence from building with geometric shapes in the preschool years to sophisticated interpretation and analysis of geometric situations in high school mathematics and above. Second, the concepts and actions of creating and then iterating units and higher-order units in the context of constructing patterns, measuring, and computing are established bases for mathematical understanding and analysis (Clements et al., 1997; Reynolds & Wheatley, 1996; Steffe & Cobb, 1988). The domain is significant to research and theory in that there is a paucity of research on the trajectories students might follow in learning this content.

Phase 2: Build an Explicit Model of Students' Knowledge Including Hypothesized Learning Trajectories

In this phase, developers build a sufficiently explicit cognitive model of students' learning that describes the processes involved in the construction of the goal mathematics concepts. Although extant models may be available, they vary in degree of specificity. Developers build these models, or fill in details of existing models, by using clinical interviews and observations to examine students' knowledge of the content domain, including intuitive ideas, and informal strategies used to solve problems. These cognitive models are then synthesized into hypothesized learning trajectories (Cobb & McClain, in press; Gravemeijer, 1999; Simon, 1995).

To continue our example, the basic structure of our model of students' knowledge of shape composition was determined by observations made in the context of early research (Sarama et al., 1996) and was refined through a research review and a series of clinical interviews and focused observations by research staff and teachers (Clements, Sarama, & Wilson, 2001).

1. *Pre-Composer.* Manipulates shapes as individuals, but is unable to combine them to compose a larger shape.

2. *Piece Assembler.* Similar to level 1, but can concatenate shapes to form pictures. In free-form “make a picture” tasks, for example, each shape used represents a unique role, or function in the picture. Can fill simple frames using trial and error (Mansfield & Scott, 1990; Sales, 1994). Uses turns or flips to do so, but again by trial and error; cannot use motions to see shapes from different perspectives (Sarama et al., 1996). Thus, children at levels 1 and 2 view shapes only as wholes and see no geometric relationship between shapes or between parts of shapes (i.e., a property of the shape).

3. *Picture Maker.* Can concatenate *shapes* to form pictures in which several shapes play a single role, but uses trial and error and does not anticipate creation of a new *geometric shape.* Chooses shapes using gestalt configuration or one component such as side length (Sarama et al., 1996). If several sides of the existing arrangement form a partial boundary of a shape (instantiating a schema for it), the child can find and place that shape. If such cues are not present, the child matches by a side length. The child may attempt to match corners, but does not possess angle as a quantitative entity, so will try to match shapes into corners of existing arrangements in which their angles do not fit. Rotating and flipping are used, usually by trial-and-error, to try different arrangements (a “picking and discarding” strategy). Thus, can complete a frame that suggests that placement of the individual shapes but in which several shapes together may play a single semantic role in the picture.

4. *Shape Composer.* Combines shapes to make new shapes or fill frames, with growing intentionality and anticipation (“I know what will fit”). Chooses shapes using angles as well as side lengths. Eventually considers several alternative shapes with angles equal to the existing arrangement. Rotation and flipping are used intentionally (and mentally, i.e., with anticipation) to select and place shapes (Sarama et al., 1996). Can fill complex frames (Sales, 1994) or cover regions (Mansfield & Scott, 1990). Imagery and systematicity grow within this and the next levels. In summary, there is intentionality and anticipation, based on shapes’ attributes, and thus, the child has imagery of the component shapes, although imagery of the composite shape develops within this level (and throughout the next levels).

5. *Substitution Composer.* Deliberately forms composite units of shapes (Clements et al., 1997) and recognizes and uses substitution relationships among these shapes (e.g., two pattern block trapezoids can make a hexagon).

6. *Shape Composite Iterater.* Constructs and operates on composite units intentionally. Can continue a pattern of shapes that leads to a “good covering," but without coordinating units of units.

7. *Shape Composer with Units of Units.* Builds and applies units of units (superordinate units). For example, in constructing spatial patterns, children extend their patterning activity to create a tiling with a new unit shape—a (higher-order) unit of unit shapes that they recognize and consciously construct;

that is, children conceptualize each unit as being constituted of multiple singletons and as being one higher-order unit (Clements et al., 1997).

The complete result of this phase is an explicit cognitive model of students' learning of mathematics in the target domain. Ideally, such models specify knowledge structures, the development of these structures, including mechanisms or processes related to this development, and trajectories that specify hypothetical routes that children might take in learning the mathematics.

Phase 3: Create an Initial Design for Software and Activities

In this phase, developers create a basic design for the software and the activities. We made the philosophical and pedagogical decision to base the *Building Blocks* project on the following approach: *Finding the mathematics in, and developing mathematics from, children's activity.* The materials should help children extend and mathematize their everyday activities, from building blocks (the second meaning to our projects' name) to art to songs and stories to puzzles. Activities should be designed based on children's experiences and interests, with an emphasis on supporting the development of *mathematical* activity. Mathematization emphasizes representing—creating models of activity with mathematical objects, such as numbers and shapes, and mathematical actions, such as counting or transforming shapes. Materials should embody these actions-on-objects in a way that mirrors what research has identified as critical *mental actions*—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks include creating, copying, and combining objects such as shapes or numbers.

It is just these cognitive building blocks that must be specified for this phase of our model. The central component of this phase of the design process is to describe the objects that will constitute the software environment and the actions that may be performed on these objects based on the model of students' learning generated in phase 2. These actions-on-objects should mirror the hypothesized mathematical activity of students. Offering students such objects and actions to be performed on these objects is consistent with the Vygotskian theory that mediation by tools and signs is critical in the development of human cognition (Steffe & Tzur, 1994). Further, designs based on objects and actions force the developer to focus on explicit actions or processes and what they will mean to the students.

Objects are, of course, a form of on-screen manipulative. Indeed, the flexibility of computer technologies allows the creation of a vision less hampered by the limitations of traditional materials and pedagogical approaches. This raises an important issue that must be addressed before we continue our example. Some early childhood educators question the benefit of using manipulatives on computer. They argue that young children benefit much more from the tactile experience of interacting with concrete manipulates. But can on-screen manipulatives still be “concrete?” One has to examine what “concrete” means. Sensory characteristics do not adequately define it (Clements & McMillen, 1996; Wilensky, 1991).

First, it cannot be assumed that children’s conceptions of the manipulatives are similar to those of adults (Clements & McMillen, 1996). For example, a student working on place value use the “ten stick” as indicating one and vice versa (Hiebert & Wearne, 1992). Second, physical actions with certain manipulatives may suggest different mental actions than those we wish students to learn. For example, researchers found a mismatch among students using the number line to perform addition. When adding five and four, the students located 5, counted “one, two, three, four,” and read the answer. This did not help them solve the problem mentally, for to do so they have to count “six, seven, eight, nine” and at the same time count the counts—6 is 1, 7 is 2, and so on. These actions are quite different (Gravemeijer, 1991). Thus, manipulatives themselves do not carry the meaning of the mathematical idea. Students must act on these manipulatives in the context of well-planned activities, and ultimately reflect on these actions. Later, we expect them to have a “concrete” understanding that goes beyond these physical manipulatives. It appears that there are different ways to define “concrete” (Clements & McMillen, 1996). We define Sensory-Concrete knowledge as that in which students must use sensory material to make sense of an idea. For example, at early stages, children cannot count, add, or subtract meaningfully unless they have actual things. They build Integrated-Concrete knowledge as they learn. Such knowledge is connected in special ways. This is the root of the word concrete—“to grow together.” What gives sidewalk concrete its strength is the combination of separate particles in an interconnected mass. What gives Integrated-Concrete thinking its strength is the combination of many separate ideas in an interconnected structure of knowledge (Clements & McMillen, 1996).

On this basis, *Building Blocks* offers children on-screen manipulative shapes as the mathematical objects. The actions include rigid transformations (slide, turn, and flip tools), duplication, and de/composition (e.g., glue and axe tools). One class of activities that involve these mathematical actions-on-objects builds on young children’s experiences with and love of the everyday activity of puzzles. Children solve outline puzzles with pattern blocks off and on the computer. Research shows this type of activity to be motivating for young children (Sales, 1994; Sarama et al., 1996). On the computer they play “Shape Puzzles” (see Figure 1), working with shapes and composite shapes as objects, creating, duplicating, positioning (with geometric motions), combining, and decomposing both individual shapes (units) and composite shapes (units).

The developers next create a sequence of instructional activities (that use objects and actions) to move students through the hypothesized learning trajectories. For the purposes of brief illustration of the essential features, only the mathematically significant basic elements are described and illustrated in Figure 2.

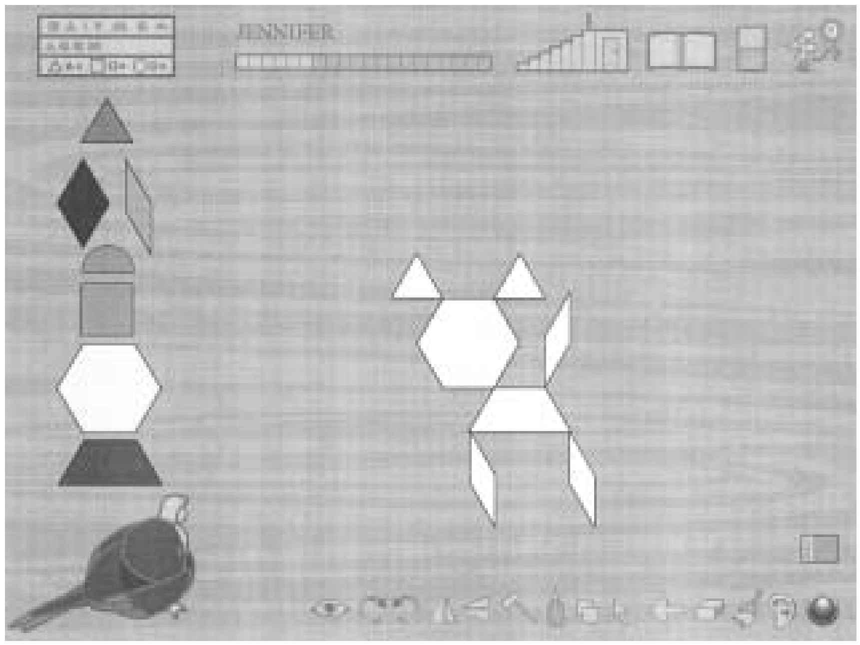


Figure 1. Shape Puzzles.

Ample opportunity for student-led, student-designed, open-ended projects must be included in the set of activities. Design activity on the part of students is frequently the best way for students to express their creativity and integrate their learning, and the computer can especially offer support for such projects (Clements, 2000). For Shape Puzzles, students design their own puzzles with the shapes; when they click on a "Play" button, their design is transformed into a shape puzzle that either they or their friends can solve.

We complete this section with two caveats. First, designs, research questions, and methodologies should remain sensitive to new possibilities. However, research indicates that technological "bells and whistles" should not become a central concern: While they can affect motivation, they rarely emerge as critical to children's learning. Instead, the critical feature is the degree to which the computer environment successfully implements education principles born from specific research on the teaching and learning of specific mathematical topics (Sarama, 2000). Second, basic research principles must be elaborated and refined by ongoing research and development work that tracks the effectiveness of specific implementations. This means that curriculum and software is not only based on research a priori. Research also must be conducted throughout the development process.

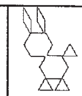
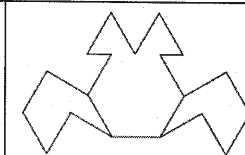
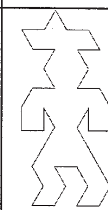
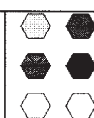
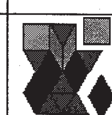
1. **Piece Assembler.** Child completes a picture given a frame that suggests the placement of the shapes, each of which plays a separate semantic role in the picture and that requires no flips or turns.		
2. **Picture Maker.** Child completes a picture given a frame that suggests the placement of the individual shapes but in which several shapes together may play a single semantic role in the picture and in which turns or flips are required. Puzzles often provide situations in which matching side lengths is a useful strategy.		
3. **Shape Composer**. The child must use given shapes to completely fill a region that consists of multiple corners, requiring selecting and placing shapes to match angles. Later tasks challenge children to fill complex frames or regions in which shape placement is ill defined, allowing for multiple solutions. These tasks require use of turning and flipping and eventually the discrimination of these.		
4. **Substitution Composer.** The child is challenged to find as many different ways as possible to fill in a frame or region, emphasizing substitution relationships (as the child is doing to the hexagons) and angle equivalence (in the finish the wallpaper activity at the far right).		

Figure 2. A sequence of tasks for Shape Puzzles.

Phase 4: Investigate the Components

Space limitations constrain our descriptions of all other phases. Phase 4 is especially interwoven with the previous one. Components of the software are tested using clinical interviews and observations of a small number of students. A critical issue concerns how children interpret and understand the screen design, objects, and actions. A mix of model (or hypothesis) testing and model generation (e.g., a microethnographic approach, see Spradley, 1979) is used to understand the meaning that students give to the objects and actions. To accomplish this, developers may use paper or physical material mock-ups of the software or early prototype versions.

A small example from the *Building Blocks* project is our research on children's initial interpretation of the actions that each icon might engender. For the decomposition of units, we had created a hammer icon. Even with minor prompting, children did not interpret this tool as breaking things apart, but instead as "nailing down" items ("It will hammer the shapes down harder") or "hammering it off" the paper or screen. We therefore created new icons (an axe "chops" shapes apart). More significant is our work with individual students using the tools. We have found that the use of these tools encourages children to become explicitly aware of the actions they perform on the shapes. Refining the tool interface for younger children while keeping the benefits researched previously is a continuing challenge.

Phase 5: Assess Prototypes and Curriculum

In this phase, the developers continue to evaluate the prototype, rendered in a more complete form. A major goal is to test hypotheses concerning features of the computer environment that are designed to correspond to students' thinking. Do their actions on the objects substantiate the actions of the researchers' model of children's mathematical activity? If not, should the model be changed, or the way in which this model is instantiated in the software? Do students use the tools to perform the actions only with prompting? If so, what type of prompting is successful? In all cases, are students actions-on-objects enactments of their cognitive operations (Steffe & Wiegel, 1994), and as models of informal mathematical activity (c.f., Gravemeijer, 1999), in the way the model posits, or merely trial-and-error or random manipulation? In general, do the microworlds engage children, mirroring their natural interactions with their environment and extending the mathematical activity? In *Building Blocks,* several of the microworlds employ everyday themes such as setting the table and making cookies. These microworlds have the advantage of authenticity (Papert, this volume, discusses setting the table as a type of microworld) as well as serving as a way for children to mathematize these activities (e.g., in setting tables, using different mathematical actions such as establishing one-to-one correspondence, counting and using numerals to represent and generate quantities in the solution of variations of the task).

Similarly, the developers test the learning trajectories and adjust them as needed. Using the cognitive model and learning trajectories as guides, and the software and activities as catalysts, the developer creates more refined models of particular students. Simultaneously, the developer describes what observed elements of the teaching and learning environment contributed to student learning. The theoretical model may involve disequilibrium, modeling, internalization of social processes, practice, and combinations of these and other processes. The connection of these processes with specific environmental characteristics and teaching strategies and student learning is critical.

With so many research and development processes happening, and so many possibilities, extensive documentation is vital. Videotapes (for later microgenetic analysis), audiotapes, and field notes are collected. This documentation should be used also to evaluate and reflect on those components of the design that were based on intuition, aesthetics, and subconscious beliefs.

In the *Building Blocks* project, we have found that children using the computer tools develop compositional imagery. Off-computer, kindergartner Mitchell gave himself the task of making multiple hexagons. He used a trial-and-error strategy, not checking to see if he had a hexagon until a shape was completed. On computer, Mitchell started making a hexagon out of triangles. After placing two, he counted with his finger on the screen around the center of the incomplete hexagon, imaging the other triangles. He announced that he would need four more. After placing the next one, he said, “Whoa! Now, three more!” The intentional and deliberate actions on the computer led him to form images (decomposing the hexagon mentally) and predict each succeeding placement. As a second example, consider Alyssa, whose work is illustrated in the first picture of step 4 (the six hexagons) in Figure 2. As Alyssa fills the hexagons, she evinces understanding of both anticipatory use of geometric motions and substitution relationships and therefore notions of area, equivalence, and congruence.

Phase 6: Conduct Pilot Tests in a Classroom

Teachers are involved in all phases of the design model. Starting with this phase, a special emphasis is placed on the process of curricular enactment (Ball & Cohen, 1996). There are two research thrusts. First, teaching experiments continue, but in a different form. The researchers conduct classroom-based teaching experiments (including what we call interpretive case studies) with one or two children. The goal is making sense of the curricular activities as they were experienced by individual students (Gravemeijer, 1994). Such interpretive case studies serve similar research purposes as teaching experiments but are conducted in a naturalistic classroom setting. Videotapes and extensive field notes are required so that students’ performance can be examined repeatedly for evidence of their interpretations and learning.

Second and simultaneously, the entire class is observed for information concerning the usability and effectiveness of the software and curriculum. Ethnographic participant observation is used heavily because we wish to research the teacher and students as they construct new types of classroom cultures and interactions together (Spradley, 1980). The focus is on how the materials are used and how the teacher guides students through the activities (for our preschool materials, child care providers and parents are also involved; classroom dynamics cannot be taken as a given). Attention is given to how software experiences reinforce, complement, and extend learning experiences with manipulatives or print (Char, 1989) as well as the diversity in the practices of the different early childhood settings.

This pilot test phase usually involves teachers working closely with the developers. The class is taught either by a team including one of the developers and the teacher, or by a teacher familiar with and intensively involved in curricula development.

In our *Building Blocks* project, several teachers in multiple settings volunteered to be a part of this phase of testing. It is important not to choose classrooms based on convenience, especially considering access to technology. We need to be able to identify what supports, both curricular and material, teachers will need to successfully and comfortably use their materials in their school environments. Results of the field testing extensively influenced the design of our software and print materials and will be published separately, which brings us to the final phase.

Phase 7: Publishing

Wider field tests and recursive re-writing of the materials are included in subsequent phases that space prohibits our describing here (but see Clements, 2002). We do wish to emphasize the difficulties and importance of publication. The software and curricula may be disseminated through a variety of channels, from commercial publishers to the Internet. As simple as this seems, this phase is not unproblematic for both curriculum/software development and research.

Regarding curriculum, negotiations and cooperation with a commercial publisher can have a substantive influence on the final software and print materials. The demands on, and of, publishers, were detailed in a previous section. Suffice it to say that these same pressures are exerted on any curriculum that is commercially published. In addition, multimedia based materials often require even more support and cooperation from publishers, and there is far less financial support for innovative software materials, especially in proportion to what is required. Therefore, there may be less freedom for developers to publish their own version of their materials. These pressures often are exerted regardless of the research base for the materials, resulting in software, originally designed to support in-depth problem solving and student evaluation of mathematical strategies and products, to shift towards activities characterized by simpler problems and feedback.

Regarding research, there are constraints to publication. Many interesting pieces of software have been created; however, the expertise developed during the production of that software has not been disseminated. Whether this is because resources are exhausted (finances, time, and emotional energy) or because there is no interest, nonpublication has a strong deleterious effect on the field of curricula development and research.

SUMMARY AND CONCLUSIONS

We believe that implementing a model of curriculum and software development such as described here is essential to building a research base for curriculum and software development as scientific enterprises, and for moving toward a time when a solid research basis is demanded of all curricula that are used widely. Presently in the United States, this is far from the case. In contrast, curriculum and software design can and should have an explicit theoretical and empirical foundation, beyond its genesis in someone's intuitive grasp of children's learning. It also should interact with the ongoing development of theory and research—reaching toward the ideal of testing a theory by testing the software and the curriculum in which it is embedded.

Although mentioned briefly, it is easy to overlook the power of *Building Blocks'* combined strategies. Research-based computer tools stand at the base, providing computer analogs to critical mathematical ideas and processes. These are used, or implemented, with activities and a management system that guides children through research-based learning trajectories (developed over years of synthesizing our own and others' empirical work). These activities-through-trajectories connect children's informal knowledge to more formal school mathematics. The result is a package that is motivating for children but, unlike "edu-tainment," results in significant assessed learning gains. In this way, *Building Blocks* has substantial potential simultaneously to develop pedagogically efficacious materials that will serve as tools of content and pedagogical reform, to conduct formative and summative evaluations at multiple levels, and to have a positive effect on the participating, low-income, school and home settings. Such synthesis of curriculum/technology development as a scientific enterprise and mathematics education research will reduce the separation of research and practice in mathematics and technology education and produce results that are immediately applicable by practitioners (parents, teachers, and teacher educators), administrators and policy makers, and curriculum and software developers.

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