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Building Blocks for early childhood mathematics

Julie Sarama, Douglas H. Clements*

Department of Learning and Instruction, University of Buffalo, The State University of New York, 505 Baldy Hall, Buffalo, NY 14260-1000, USA

Abstract

Building Blocks is a National Science Foundation-funded project designed to enable all young children to build a solid foundation for mathematics. To ensure this, we used a design and development model that drew from theory and research in each phase. Our design process is based on the assumption that curriculum and software design can and should have an explicit theoretical and empirical foundation, beyond its genesis in someone's intuitive grasp of children's learning. It also should interact with the ongoing development of theory and research-reaching toward the ideal of testing a theory by testing the software and curriculum in which it is embedded. Our model includes specification of mathematical ideas (computer objects or manipulatives) and processes/skills (software "tools" or actions) and extensive field-testing from the first inception through to large summative evaluation studies. The initial field test results indicate that such an approach can result in significant assessed learning gains consistent with the new *Principles and Standards for School Mathematics* of the National Council of Teachers of Mathematics.

Keywords: Counting; Critical thinking; Curriculum; Developmental/learning continuum; Developmentally appropriate practice; Early childhood; Geometry; Instruction; Low-income children; Materials; Mathematics; Measurement; National Association for the Education of Young Children (NAEYC); NCTM; NSF; Number; Patterns; Preschool; Shape; Spatial relations; Spatial sense; Standards; Teachers

Building Blocks is a National Science Foundation-funded project designed to enable all young children to build a solid foundation for mathematics. To ensure this, we used a design and development model that drew from theory and research in each phase. Most developers claim a research basis for their materials, but these claims are often vacuous or incomplete (Sarama & Clements, in press). *Building Blocks* is research-based in several fundamental ways. Our design process is based on the assumption that curriculum and software design can and should have an explicit theoretical and empirical foundation, beyond its genesis in someone's intuitive grasp of children's learning. It also should interact with the ongoing development of theory and research—reaching toward the ideal of testing a theory by testing the

* Corresponding author. Tel.: +1-716-636-2455; fax: +1-716-645-3161.
E-mail address: elements@buffalo.edu (D.H. Clements).

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software and the curriculum in which it is embedded. Our model includes specification of mathematical ideas (computer objects or manipulatives) and processes/skills (software "tools" or actions) and extensive field-testing from the first inception through to large summative evaluation studies (Clements, 2002; Clements & Battista, 2000; Sarama & Clements, in press). The next section briefly describes the phases of our design process model. We illustrate these phases with the first of several preschool products that will be produced by the *Building Blocks* project (Clements & Sarama, 2003).[1]

1. The *Building Blocks* model for the development of research-based curricula

1.1. Phase 1: draft initial goals

The first phase begins with the identification of significant domains of mathematics. One of the reasons underlying the *Building Blocks* name was our desire that the materials emphasize the development of basic *mathematical building blocks*—ways of knowing the world mathematically—organized into two areas: (a) spatial and geometric competencies and concepts and (b) numeric and quantitative concepts. Research shows that young children are endowed with intuitive and informal capabilities in both of these areas (Bransford, Brown, & Cocking, 1999; Clements, 1999; Ginsburg, Klein, & Starkey, 1998).

1.2. Phase 2: build an explicit model of children's thinking, including learning trajectories

In this phase, developers build an explicit cognitive model of students' goal mathematics concepts. These cognitive models are synthesized into hypothesized learning trajectories (LT) (Cobb & McClain, in press; Simon, 1995). As an example, the following are selected levels, with brief illustrations, from one number LT—addition.

1.2.1. Non-verbal addition

After watching two objects, then one more placed under a cloth, children choose or make collections of three to show how many are hidden in all.

1.2.2. Small number addition

Children solve simple word problems with sums to about five, usually by subitizing (instant recognition of small collections) or using a "counting all" strategy.

1.2.3. Find result

Children solve "result unknown" problems by direct modeling—"separating from" for subtraction or counting all for addition. For example, young children might be told they have three blocks, and got four more blocks, and asked how many they would have in all.

[1] These products were designed for preschool, but the software is also appropriate for kindergarten (Clements & Sarama, 2003). The entire set of materials includes literacy and all developmental areas (Schiller, Clements, Sarama, & Lara-Alecio, 2003). A standalone version of the mathematics materials is "in press." In addition, we are presently producing a different, but related, set of Pre-K to grade 2 materials.

1.2.4. Find change

Children solve "change unknown" word problems by direct modeling. For example, they might "add on" to answer how many more blocks they would have to get if they had four blocks and needed six blocks in all.

1.2.5. Counting on

Children continue developing their counting methods even further, often using objects. For example, children may add $3 + 2$ by counting on, "threeeee . . . 4 [putting up one finger], 5 [putting up a second finger]. Five!" Such counting requires conceptually embedding the 3 inside the total, 5.

This continues through levels of *Counting Strategies*, *Derived Combinations*, and beyond. The result of this phase is an explicit cognitive model of students' learning of mathematics in the target domain, including knowledge structures, the development of these structures, including mechanisms or processes related to this development, and LT that specify hypothetical routes that children might take in learning the targeted mathematics competencies (space constraints prohibit full description—see Clements, Sarama, & DiBiase, 2004).

1.3. Phase 3: create an initial design for the curriculum

In this phase, developers create a basic design for the software and the activities. First, a pedagogical stance is chosen. Based on theory and research on early childhood learning and teaching (Bowman, Donovan, & Burns, 2001; Clements, 2001), we determined that *Building Blocks*' basic approach would be *finding the mathematics in, and developing mathematics from, children*'s *activity*. The materials should help children extend and mathematize their everyday activities, including art, songs, stories, puzzles and building blocks (another reason for the *Building Blocks* name). Activities should be based on children's experiences and interests, with an emphasis on supporting the development of mathematical activity. With computers, manipulatives (including everyday objects), and print materials, the curriculum should encourage children to *represent* mathematical ideas. They create models of their activity with mathematical *objects*, such as numbers and shapes, and mathematical actions, such as counting. The materials should embody these actions-on-objects in a way that mirrors what research has identified as critical mental actions—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks include creating, copying, and combining objects such as shapes or numbers.

For example, *Building Blocks* offers children onscreen manipulative objects and shapes as the mathematical objects. The actions on objects include counting, adding to, taking away, combining, and so forth. The actions on shapes include rigid transformations (slide, turn, and flip tools), duplication, and de/composition (e.g., glue and axe tools). We next create a sequence of instructional activities that use these objects and actions to move students through the LTs.

To continue our addition example, we designed three off- and on-computer activity sets, each with multiple levels: Double Trouble, Dinosaur Shop, and Number Pictures. Mrs. Double has twins who always want the same number of chocolate chips on their cookies. The early levels of Double Trouble use this scenario to teach matching collections and various counting skills. At higher levels, children add. For example, Mrs. Double may hide two, then one more chip under a napkin, and the child is asked to make the second cookie have the same number of chips—the *Non-Verbal Addition* level of our LT (Fig. 1). The teacher conducts the same activity with children using colored paper cookies and brown buttons for "chips."

Fig. 1. Level 3 of Double Trouble, in which children decorate cookies and practice counting and adding by using specific numbers of chips.

At the *Small Number Addition* level, the teacher introduces a socio-dramatic play area, the dinosaur shop, and encourages children to count and add during their play. Children help run a software dinosaur shop as well. For example, level 3 develops what *research indicates is a foundational cognitive building block of addition*: Mentally combining two groups into one larger group—even before knowing the specific number. A customer orders two types of items, and children label these two collections in two separate boxes (e.g., two red triceratops and three yellow brontosauruses). The customer asks for one box and children move all the dinosaurs into a new box (Fig. 2). They then label the new box with the sum (the *Find Result* level).

The next level of the LT is *Find Change*. Here, Mrs. Double (or the teacher, working with small groups manipulatives) puts out three chips and asks the child to "make it six," encouraging adding on (or *Counting On*, for children who can use more sophisticated strategies). A similar level exists in the Dinosaur Shop activities.

Ample opportunity for student-led, student designed, open-ended projects are included in each set of activities. Problem posing on the part of students is an effective way for students to express their creativity and integrate their learning, and the computer is especially apt at offering support for such projects (Clements, 2000). For example, children can make up their own problems with cookies and chips, or dinosaurs and boxes. As another example of a different activity, children design their own "Number Pictures" with shapes and see the resulting combination (Fig. 3; as always, it is also conducted off computer). This activity also illustrates the integration of counting, addition, geometry, and processes such as representation.

1.4. Phase 4: investigate the components

In this phase, developers test components of the curriculum and software using clinical interviews and observations of a small number of students. A critical issue concerns how children interpret and

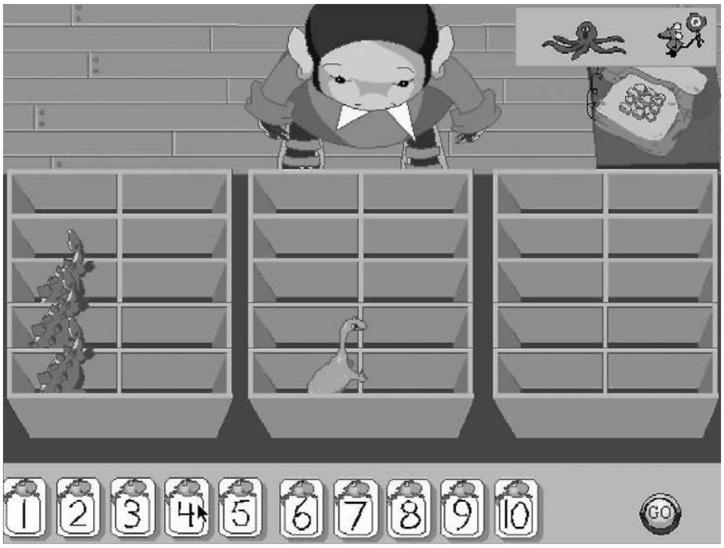

Fig. 2. Level 3 of Dinosaur Shop, in which children develop basic counting and addition skills as they label boxes and fill orders in a toy shop.

understand the objects, actions, and screen design. A mix of model (or hypothesis) testing and model generation (e.g., a microethnographic approach, see Spradley, 1979) is used to understand the meaning that students give to the objects and actions. To accomplish this, developers may use paper or physical material mock-ups of the software or early prototype versions.

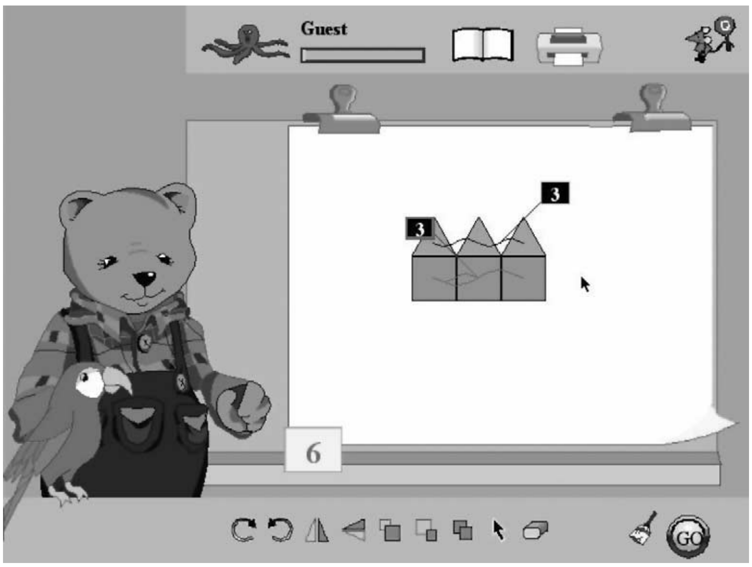

Fig. 3. Level 1 of Number Pictures in which children make number pictures or designs from a specified number of shapes.

1.5. Phase 5: assess prototypes and curriculum

In this phase, developers continue to evaluate the prototype, rendered in a more complete form. A major goal is to test hypotheses concerning features of the curriculum that are designed to correspond to students' thinking. Do their actions on the objects substantiate the actions of the researchers' model of children's mathematical activity? If not, what should be changed, the model (and LT) or the way it is instantiated? Do students use the tools to perform the actions only with prompting? If so, what type of prompting is successful?

For *Building Blocks'* addition activities, we found that we did not need to adjust LTs (although we did for other LTs). We did determine effective prompts to incorporate into each level of each activity. We found that following any incorrect answer, effective prompts first ask children to try again such prompts then provide one or more increasingly specific hints, and eventually demonstrate an effective strategy and the correct answer. Asking children to go slowly and try again was successful in a large number of cases; throughout, we strove to give "just enough help" and encourage the child to succeed as independently as possible. However, the specific hints had to be fine-tuned for each activity. Given the activities and prompts, students employed the thinking strategies we had desired.

1.6. Phase 6: conduct pilot test in a classroom

Teachers are involved in all phases of the design model. Starting with this phase, a special emphasis is placed on the process of curricular enactment (Ball & Cohen, 1996). There are two research thrusts. First, teaching experiments continue, but in a different form. The developers conduct classroom-based teaching experiments with one or two children at a time. The goal is making sense of the curricular activities as individual students experienced them (Gravemeijer, 1994). Second and simultaneously, the developers observe the entire class for information concerning the usability and effectiveness of the software and curriculum.

We found that the activities did provide multiple opportunities to perform simple addition and subtraction. For example, as Geri played with Janelle and Andre in the classroom's dinosaur shop, she filled many orders. This involved reading a numeral on a card and counting out the correct quantity for her customers and collecting the correct amount of play money. Eventually, Janelle wanted to "trick" Geri and gave her two cards, a 2 and a 5 (a problem from the *Find Result* level). The teacher suggested Geri give Janelle 2 of one kind and 5 of another. She carefully counted out the two piles, put them together and counted the total. She then asked Janelle for $7. Pre- and posttesting revealed small but significant pre-post gains on adding and subtracting small numbers (totals <4) without objects and large gains on adding with objects (Sarama, 2004).

1.7. Phase 7: conduct pilot tests in multiple classrooms

In this phase developers gradually expand the range of size and scope to studies of what can actually be achieved with typical teachers under realistic circumstances. Innovators often conduct too little research at this level. Innovative materials too often provide less support than the textbooks with which teachers are accustomed, even when they are teaching familiar topics. Developers need to understand what the curriculum should include to fully support teachers of all levels of experience and enthusiasm for adopting the new curriculum.

To date, we have conducted summative research in only four classrooms. To evaluate the quantitative results, we used the accepted benchmarks of 0.25 or greater as an effect size that has practical significance (i.e., is educationally meaningful), 0.5 for an effect size of moderate strength, and 0.8 as a large effect size (Cohen, 1977). Addition had an effect size greater than 0.8.

Along with colleagues Prentice Starkey and Alice Klein, we have been funded by the Department of Education to conduct a "Preschool Curriculum Evaluation Research" (PCER) project to evaluate the curriculum with a wider range of teachers, including random assignment of teachers to experimental and control groups. In a similar vein, we believe there may be no more challenging issue than that of effectively scaling up such an educational intervention with the diverse population who teaches Pre-K and the diversity of program structures in the early childhood system in the U.S. We need to investigate whether we can effectively *scale up* the implementation of early childhood mathematics curricula to a far larger number of teachers, schools, and school systems. Again with our colleagues, We are conducting such a research project funded by the Interagency Educational Research Initiative (NSF, DOE, and NICHHD).

2. Emergent questions for future research

The final phases of our development process have not yet been completed. Therefore, it will not be surprising that we believe the most immediate research need is to investigate whether curricula such as those described in this special issue are effective when implemented by a larger number of teachers. What other types of questions follow from our approach and initial research findings? We believe that our qualitative research indicates that our goal for the early years should be *mathematical* literacy, not "numeracy". The latter usually is (mis)understood as dealing mainly with numbers. Our initial evidence indicates that geometry and patterning are foundational for mathematics learning. They are important in and of themselves. They build on the interests and competencies of young children. Finally, they support the learning of other mathematical topics, such as number (from counting the sides of shapes to seeing numbers in rows and columns).

In addition, we believe that a complete mathematics program may contribute to later learning of other subjects, especially literacy. Many of the recent mathematics projects also have reported that mathematics aids the development of literacy. We need empirical evidence that goes beyond the existing (mostly anecdotal) reports. In particular, we have reason to believe that geometry is fundamental in this arena as well. Previous research reports that preschool children who were taught geometry not only make gains in geometric and spatial skills but also show pronounced benefits in the areas of arithmetic and writing readiness (Razel & Eylon, 1990). Our own research substantiates that children are better prepared for all school tasks when they gain the thinking tools and representational competence of geometric and spatial sense.

Another question is the focus on numbers. Other programs, notably that of Ginsburg, Greenes, and Balfanz, illustrate that children love large numbers. We concur, but have found the most benefit when we spend most of our instructional time on deep learning of small numbers (e.g., to 10 or 20). An optimal balance must be found.

As have others, we have found that children are more competent than most programs assume. However, we have also uncovered profound variability. This indicates that "starting where children are" and ensuring they "develop with the program" are critical for deep learning. Large group instruction often hides the needs of individuals. Research that identifies effective ways of combining group work with individual assessment and teaching will make a major contribution.

Technology can also make a significant contribution, if it is *fully* based on research *and* integrated into the curriculum. We need more research that identifies its specific contribution. For example, our past and present research indicates that computers can help individualize, but also help *mathematize* for children. How can we help teachers achieve such benefits, extending them to more topics and more children?

Finally, we have found that basing the curriculum on learning trajectories is even more important than we originally assumed. They helped sequence activities and were critical for allowing our software to provide correlated, individualized activities. In addition, we found that teachers who understood the LT were more effective in teaching small groups and encouraging informal, incidental mathematics *at an appropriate and deep level.* This is consistent with previous research on teachers who develop profound knowledge of mathematics in their students (Cobb, 2001; Fuson, Carroll, & Drueck, 2000; Simon, 1995; Stigler & Hiebert, 1999)—they understand LTs for specific topics. We have found this the most powerful way to help our teachers understand children's development, conduct observational assessment, teach, and appreciate the worth of a curriculum. Can this be done on a large scale? What level of detail in the LT is ideal for this type of work?

3. Summary

We believe in the power of *Building Blocks'* combined strategies. Research-based computer tools stand at the base, providing computer analogs to critical mathematical ideas and processes. These are implemented with activities and a management system that guides children through fine-grained, research-based learning trajectories. These activities-through-trajectories—on and off the computer—connect children's informal knowledge to more formal school mathematics. The result is a package that is motivating for children, but is also comprehensive in that it includes both exploratory environments that include specific tasks and guidance, building concepts and well-managed practice building skills, a full set of critical curriculum components, and full range of mathematical activities. The initial field test results indicate that such an approach can result in significant assessed learning gains consistent with the new *Principles and Standards for School Mathematics* of the National Council of Teachers of Mathematics.

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