The volatility puzzle of the low-risk anomaly[*]

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Abstract

The volatility of betting-against-beta (BAB) and -idiosyncratic volatility (BAV) factors negatively forecasts their respective Sharpe ratios and abnormal returns. This predictability causes significant performance gains from volatility timing these factors and provides new time-series evidence on leading theories of the low-risk anomaly. Consistent with the limits-to-arbitrage theory, we show that the abnormal returns of the volatility-managed BAV strategy are concentrated in overpriced stocks. However, controlling for mispricing, arbitrage risk, lottery demand, and multiple risk factors has no effect on the timing benefits of BAB. We further show that the leverage constraints model predicts a counterfactual positive relation between volatility and subsequent BAB Sharpe ratios, and highly active institutions shift from high- to low-beta stocks as volatility increases, suggesting their demand contributes to the abnormal returns of BAB. Overall, the predictive power of volatility challenges our current understanding of the low-risk anomaly.

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"We keep regurgitating the data to find yet one more variation of the size, value, or momentum anomaly, when the Mother of all inefficiencies may be standing right in front of us—the risk anomaly."

—Robin Greenwood quoted in Ang (2014), page 332.

1. Introduction

Understanding the relationship between risk and return is perhaps the most central pursuit of asset pricing. One of the oldest and most well-known facts about this relationship is that assets with low risk, measured by beta or volatility, earn positive abnormal returns relative to the CAPM of Sharpe (1964) and Lintner (1965), with the opposite result holding for high-risk stocks, a phenomenon referred to as the low-risk anomaly. This anomaly dates to at least Friend and Blume (1970) and Black, Jensen, and Scholes (1972) but it remains robust in the fifty years of data after these seminal studies and its cause is still actively debated, lacking a single universally accepted explanation (see, e.g., Ang, Hodrick, Xing, and Zhang, 2006; Frazzini and Pedersen, 2014; Liu, Stambaugh, and Yuan, 2018; Asness, Frazzini, Gormsen, and Pedersen, 2020). In addition to academic interest, low-risk strategies have experienced "massive capital inflows" in the words of Novy-Marx and Velikov (2021) and provide a foundation for dozens of exchange-traded funds.

In this paper, we take a novel approach to evaluating leading explanations of the low-risk anomaly based on their ability to explain the conditional performance of "betting-against-risk" (BAR) factors that buy low-risk stocks and short high-risk stocks. Moreira and Muir (2017), Cederburg, O'Doherty, Wang, and Yan (2020), and Barroso, Detzel, and Maio (2021) show that leveraging BAR factors inversely with their lagged realized volatility produces managed portfolios that earn significant abnormal returns relative to their unmanaged counterparts. These volatility-timing benefits reflect the fact that Sharpe ratios and risk-adjusted returns of BAR factors are relatively high when their ex-ante volatility is relatively low.

We consider four leading candidate explanations for the low-risk anomaly: leverage constraints (Black, 1972; Frazzini and Pedersen, 2014), limits to arbitrage (Stambaugh, Yu, and Yuan, 2015; Liu et al., 2018), lottery preferences (Bali, Cakici, and Whitelaw, 2011; Bali, Brown, Murray, and Tang, 2017), and risk factors missing from the CAPM (Schneider, Wagner, and Zechner, 2020; Novy-Marx and Velikov, 2021).

The leverage constraints theory, first proposed by Black (1972) and expanded by Frazzini and Pedersen (2014), posits that leveraged-constrained investors with relatively low risk aversion will bid up the prices of high-beta stocks because of their high expected returns, thereby rendering their CAPM alphas negative. We compare several calibrations of Frazzini and Pedersen's equilibrium model that generate different levels of BAB volatility. These calibrations show that the Sharpe ratios of both BAB and the market portfolio should increase with these factors' respective volatilities, the opposite of the results found in the data. Intuitively, these counterfactual predictions arise from the key assumption in the model that all investors are risk averse and therefore demand a higher price of risk when volatility increases.

Stambaugh et al. (2015) and Liu et al. (2018) propose the limits-to-arbitrage theory of the low-risk anomaly. The former argues that the negative cross-sectional relationship between idiosyncratic volatility (IVOL) and returns arises from two common asset-pricing assumptions. First, IVOL represents arbitrage risk that discourages sophisticated traders from eliminating abnormal returns (see, e.g., Pontiff, 2006). Second, traders devote less capital to exploiting overpricing than underpricing. As a result, the effect of IVOL should be greater on overpricing than underpricing, yielding a negative abnormal return on the portfolio of all high-IVOL stocks. Indeed, using the Stambaugh, Yu, and Yuan (2012) mispricing measure, Stambaugh et al. (2015) show that stocks with high IVOL only underperform those with low IVOL in overpriced segments of the market, with the opposite pattern holding in underpriced stocks. Liu et al. (2018) expand on these results, showing that the beta anomaly follows from the strong cross-sectional correlation between IVOL and beta. To determine whether the limits-to-arbitrage theory is consistent with the performance of volatility-managed BAR portfolios, we first double sort stocks based on the mispricing measure and beta or IVOL. We then construct BAR factors within each mispricing group along with one that controls for the cross-sectional effects of mispricing by going long low-risk stocks and short high-risk stocks, while seeking to match the mispricing of stocks bought and sold.

Consistent with the limits-to-arbitrage theory, we find that the volatility-managed betting-against-IVOL (BAV) factor constructed with overpriced stocks earns abnormal returns relative to its unmanaged counterpart but the same does not hold with any other mispricing segment or for the BAV strategy that controls for the cross-sectional effects of mispricing. In contrast, the abnormal returns of the volatility-managed BAB factors with respect to their unmanaged counterparts are statistically indistinguishable across mispricing segments and significant for BAB factors that control for mispricing. We further show BAB factors that control for IVOL in their construction also experience significant performance improvements from volatility management. Thus, the volatility-timing benefits of the BAV strategy are consistent with the limits-to-arbitrage theory but the same is not true for the BAB factors. These results are therefore inconsistent with the hypothesis that the beta anomaly arises from the cross-sectional correlation between beta and IVOL.

Bali et al. (2011) and Bali et al. (2017) posit that demand for lottery-like payoffs explains the negative abnormal returns of high-risk stocks. The latter measure lottery demand using a variable called MAX, which is the average of the five highest daily returns over the preceding month, and show that MAX subsumes the cross-sectional relationship between beta and returns. The former show that a similar variable explains the IVOL anomaly. Asness et al. (2020) scale MAX by standard deviation of returns to create a measure of lottery demand called SMAX that is not mechanically related to volatility. Hypothetically, lottery demand could explain the volatility-timing benefits of the BAR factors, though it must be the case that this demand is high when BAR volatility is low. Similar to our limits-to-arbitrage tests, we form BAR factors that control for MAX and SMAX in their construction, and show they exhibit even stronger alpha and Sharpe ratio improvements from volatility management than the baseline BAR factors. Thus, the lottery preferences theory fails to explain the low-risk anomaly conditional on volatility.

Several studies find that multifactor asset-pricing models explain the abnormal returns of the low-risk anomaly. Novy-Marx and Velikov (2021) show that the Fama and French (2018) six-factor model (FF6) prices a BAB factor based on Frazzini and Pedersen (2014)'s definition of beta and Fama and French (2016) show a similar result for portfolios formed on IVOL. We confirm these results and extend it to a BAB factor based on the betas of Liu et al. (2018). However, we also show that the FF6 fails to explain BAR returns conditional on lagged volatility. In low-volatility states, BAR Sharpe ratios more than double and their alphas become economically large (about 70 basis points per month) and statistically significant. At the same time, the factors loadings that explain BAR returns unconditionally all shrink significantly, or even flip signs. Said differently, the most anomalous returns on BAR factors occur precisely when they require bearing the least risk to earn. Schneider et al. (2020) show that coskewness risk can explain the returns on BAR portfolios based on IVOL and Frazzini and Pedersen (2014) betas. We confirm that coskewness betas of BAB portfolios are negative, however, they are small and insignificant for the BAB factor based on the Liu et al. (2018) definition of beta. Moreover, no BAR factor exhibits a significantly different coskewness beta between low- and high-volatility states. Overall, the multifactor models fail to explain the performance of BAR factors conditional on volatility.

To shed further light on the conditional performance of the BAR factors, we compare their Sharpe ratios over four "2x2" sub-sample periods based on three pairs of predictors. The first pair is the Baker and Wurgler (2007) sentiment index and BAR volatility. Consistent with Stambaugh et al. (2015) and Antoniou, Doukas, and Subrahmanyam (2016), we find that BAR Sharpe ratios are higher when sentiment is "optimistic", consistent with these studies' interpretation that, in such times, naive investors overvalue high-risk stocks. However, sentiment does not impact the volatility-timing benefits of the BAR portfolios; their Sharpe ratios are highest when volatility is low, regardless of sentiment. The second pair of predictors is the systematic and idiosyncratic components of BAR volatility, both defined relative to the Fama and French (2018) six-factor model. Neither component dominates, with Sharpe ratios increasing as either decreases. The third pair of predictors is the realized volatilities of the BAR factor and market portfolio. Again, Sharpe ratios increase as either measure of volatility decreases.

For the abnormal returns of BAB to be negatively related to lagged volatility, it must be the case that high-beta stocks become less "overvalued" relative to low-beta stocks as the volatility of BAB increases. We next investigate whether trading by institutional investors can explain this dynamic relationship. Indeed, we find that, as BAB volatility increases, "short-term" institutions, i.e., those the in highest tercile of turnover, decrease their holdings of stocks with high betas and increase their holdings of stocks with low betas. Thus, these institutions as a group trade the wrong direction to benefit from timing the volatility of BAB, which in turn suggests their demand contributes to this anomaly, especially considering that the most active arbitrageurs such as hedge funds would generally be classified as "short-term" institutions. We cannot rule out other interpretations, however, this pattern is consistent with a nascent literature arguing that widely used performance evaluation contracts incentivize institutional managers to overweight high-beta stocks, even if their alphas are negative, because their high average returns help to beat unit-beta benchmarks like the S&P 500 (e.g., Baker, Bradley, and Wurgler, 2011 and Christoffersen and Simutin, 2017). The same incentives also penalize high tracking error volatility and so, as BAB volatility increases, managers have the incentive to tilt from their positions in high-beta stocks towards their benchmarks, requiring them to sell high-beta stocks and buy low-beta ones. We find no correlation between BAV volatility and institutional holdings of high-IVOL stocks, which is also consistent with the benchmark-incentives explanation because, unlike high-beta stocks, those with high-IVOL have low average returns and therefore do not help managers beat their benchmarks. Regardless of explanation, future work attempting to explain the beta anomaly must not only contend with the fact that it is only profitable in low-risk times, but also that short-term institutions trade in the wrong direction at the margin to profit from timing this pattern.

Overall, the results in this paper challenge current explanations from the extensive literature on the low-risk anomaly. Our study is most closely related to Asness et al. (2020), who examine the compatibility of the low-risk anomaly with essentially the same leading explanations as we do. Their key innovation is creating BAR factors that separate the cross-sectional effects of the systematic risk in beta from those of idiosyncratic volatility. In contrast, our key innovation focuses on the time series, which leads us to very different conclusions. They find that multiple theories contribute to explaining the low-risk anomaly while we find that none of these theories are consistent with the conditional performance of all BAR factors. More broadly, our results show that five decades worth of attempts to explain a single anomaly fail to produce a single theory that can explain this anomaly's returns conditional on volatility. This truism highlights the importance of not ignoring conditioning information in asset pricing, which is still widely done in practice.

This paper proceeds as follows. Section 2 specifies our data sources and the construction of our BAR portfolios. Section 3 presents our main results, evaluating leading theories of the low-risk anomaly. Section 4 compares predictors and presents the institutional ownership results. Section 5 concludes.

2. Betting-against-risk factors and other data

In this section, we describe our data and the construction of our BAR factors along with their volatility-managed counterparts.

2.1. Data

2.1.1. BAR portfolios

We construct three BAR factors, two based on beta and one based on IVOL. The first BAB factor, denoted FP, is based on the beta measure of Frazzini and Pedersen (2014), "FP beta", which is computed based on separate estimations of correlations and volatilities. The correlation, $\hat{\rho}_{i,t}$, of the return on stock i in month t, with that of the market is estimated from the previous 5 years of overlapping three-day log returns. Volatilities of the stock and the market ($\hat{\sigma}_{i,t}$ and $\hat{\sigma}_{M,t}$, respectively), are estimated with one year of daily returns. Then, the FP beta is given by $\hat{\beta}_{FP,i,t} = \hat{\rho}_{i,t} \times \hat{\sigma}_{i,t}/\hat{\sigma}_{M,t}$. At the beginning of each month, we sort all stocks in CRSP into value-weighted decile portfolios based on their $\hat{\beta}_{FP,i,t}$, denoting the return on the low-beta (high-beta) portfolio as $ret_{L,t}$ ($ret_{H,t}$). Following Frazzini and Pedersen (2014) and Asness et al. (2020), we target beta neutrality by defining FP as:

$$\text{FP}_t = (1/\beta_{L,t-1})(ret_{L,t} - rf_t) - (1/\beta_{H,t-1})(ret_{H,t} - rf_t), \tag{1}$$

where $\beta_{L,t-1}$ ($\beta_{H,t-1}$) denotes the value-weighted average of the $\hat{\beta}_{FP,i,t}$ across stocks in the low-beta (high-beta) portfolio, and rf_t denotes the risk-free rate. To prevent the possibility of extreme weights in these portfolios, we bound the ex ante beta estimates of each leg of the portfolio between 0.5 and 2. Our second BAB factor, denoted LSY, is defined similarly to FP, but uses the beta measure, "LSY beta", of Liu et al. (2018). The LSY betas are based on CAPM regressions estimated each month using the prior 60 monthly returns (36 month minimum), applying a Dimson (1979) correction for nonsynchronous trading and the Vasicek (1973) shrinkage procedure to mitigate estimation error.

We use two different methods to estimate betas for robustness. The FP betas are a natural choice for comparability with prior literature as evidenced by the fact that, as of this writing, Frazzini and Pedersen (2014) is the fourth most downloaded article from *Journal of Financial Economics*. Liu et al. (2018) provide a thorough recent discussion of different methods to estimate betas and their implications for our understanding of the beta anomaly. Using a metric based on CAPM fit, they show that LSY betas have the best out-of-sample performance in forecasting future betas relative to common alternatives, including FP betas.

Our construction of FP differs in one regard from the factor proposed by Frazzini and Pedersen (2014), denoted BAB_{AQR}; they use rank-weighting instead of value-weighting in portfolio construction. Novy-Marx and Velikov (2021) observe that the construction of BAB_{AQR} has three non-standard features that materially impact its performance: i) the use of different and time-varying leverage in the long and short legs, ii) a new estimation procedure for betas, and iii) rank-weighting stocks in each leg of the portfolio instead of value-weighting (as typically done, say, for the portfolios available in Kenneth French's data library). They show that the new beta estimation method does not significantly impact the unconditional performance of BAB, but rank-weighting (iii) combined with leverage (i) causes BAB_{AQR} to place extreme weight on micro- and nano-caps, stating specifically: "For each dollar invested in [BAB_{AQR}], the strategy commits on average \$1.05 to stocks in the bottom 1% of total market capitalization." This implies that BAB_{AQR} is a strategy typical institutional investors would find hard to implement. Motivated by these concerns, we build more conventional value-weighted BAB portfolios. We keep the leverage in BAB factors, however, for two exposition-related reasons, both based on the fact high- and low-beta stocks earn similar average returns but very different CAPM alphas. Leveraging to maintain market neutrality translates the positive alphas earned by unleveraged BAB factors into positive average returns earned by the leveraged factors. Without this feature, comparisons based on Sharpe ratios, for example, would be very difficult. This feature also helps maintain comparability with the literature on volatility management, which focuses on factors that earn positive average returns (see, e.g., Barroso and Santa-Clara, 2015b and Moreira and Muir, 2017).

Following Hou, Xue, and Zhang (2020), we obtain value-weighted IVOL-decile portfolios from Lu Zhang's online data library, denoting their returns as IV_1 (Low volatility), IV_2,... IV_{10}.[1]. They estimate IVOL for each stock and month via regressions of daily stock returns on returns on the Fama and French (1993) three factors, as in Ang, Hodrick, Xing, and Zhang (2006).[2] We construct our betting-against-IVOL factor as:

$$\text{BAV}_t = \text{IV}_{1,t} - \text{IV}_{10,t}. \tag{2}$$

2.1.2. Other data

Return data for the risk-free rate and the six-factor asset-pricing model of Fama and French (2018) (FF6) come from the website of Kenneth French.[3] FF6 includes the market (RMRF) and value (HML) factors of Fama and French (1993), as well as factors based on size (SMB), robust (high)-minus-weak profitability (RMW), conservative (low)-minus-aggressive (high) real investment (CMA), and momentum (UMD) following Carhart (1997). RMW and CMA were added in Fama and French (2015), while Fama and French (2016, 2018) add the momentum factor.[4]

[1] `www.theinvestmentcapm.com`

[2] Bali, Engle, and Murray (2016) and Detzel, Duarte, Kamara, Siegel, and Sun (2019) note that this is the most standard definition of idiosyncratic volatility in the literature and remains robust in the pre- and post-Ang et al. (2006) samples, 1926–1963 and 2001–2016, respectively.

[3] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

[4] The factors in the FF6 reflect previously documented cross-sectional patterns in equity returns based on size (Banz, 1981), value (Basu, 1983; Bondt and Thaler, 1985; and Rosenberg, Reid, and Lanstein, 1985), profitability (Haugen and Baker, 1996; Fama and French, 2006; and Novy-Marx, 2013), investment or asset growth (Titman, Wei, and Xie, 2004; Lyandres, Sun, and Zhang, 2007;

We use the stock-level mispricing measure of Stambaugh et al. (2012), Stambaugh et al. (2015), and Stambaugh and Yuan (2017), obtained from the website of Robert Stambaugh.[5] They construct a stock's mispricing measure each month as the average of the stock's rankings with respect to 11 variables associated with prominent anomalies relative to the Fama and French (1993) three-factor model (FF3). For each anomaly variable, they assign a ranking percentile to each stock reflecting the cross-sectional sort on that variable. High ranks correspond to low estimated alpha. A stock's mispricing measure in a given month is the simple average of its ranking percentiles across the anomalies such that, the higher is this average ranking, the more overpriced is the stock relative to others in the cross-section. Due to availability of this data, our sample period is from July 1967 to December 2016.

Stock-level (total) institutional ownership, IO, is defined to be the percentage of shares owned by institutional owners and comes from Thomson Financial 13(f) Institutional Holdings at the quarterly frequency since 1980q1. Following Yan and Zhang (2009), at the beginning of each quarter, we sort institutional investors into terciles based on their portfolio turnover over the past four quarters (details can be found in their Section 1.2). Those in the highest-turnover tercile are defined to be "short-term" institutions and those in the lowest tercile are defined to be "long-term" institutions.

2.2. Volatility-managed factors

Recent studies find that volatility-managed versions of many asset-pricing factors produce significant alphas and Sharpe ratio gains relative to their unmanaged counterparts. Ang (2014), Barroso and Santa-Clara (2015b), Daniel and Moskowitz (2016), Barroso et al. (2021), and Eisdorfer and Misirli (2020) demonstrate these results for the market portfolio, momentum, the beta anomaly, and the distress anomaly. Moreira and Muir (2017), Cederburg et al. (2020), and DeMiguel, Martın-Utrera, and Uppal (2021) expand the finding to a much broader set of equity factors.[6] Volatility-managed factors increase leverage when risk was recently low, and

Xing, 2007; and Cooper, Gulen, and Schill, 2008), and momentum (Levy, 1967; Jegadeesh and Titman, 1993; and Fama and French, 1996).

[5] `http://finance.wharton.upenn.edu/~stambaug/`

[6] The benefits of volatility timing the beta anomaly were first documented by the 2016 version of Barroso et al. (2021). See also Kirby and Ostdiek (2012), who consider volatility timing strategies for portfolio allocations across stock portfolios. Much of this literature follows from Fleming, Kirby, and Ostdiek (2001, 2003), and Marquering and Verbeek (2004), who demonstrate large utility gains from volatility timing allocations across several asset classes. Banerjee, Doran, and Peterson (2007) also uses implied volatility to forecast abnormal returns of portfolios sorted on beta.

deleverage when risk increases. Their performance exploits the fact that, for the vast majority of asset-pricing factors, volatility is highly persistent from month-to-month while returns are close to unpredictable.

Following Barroso and Santa-Clara (2015b), we use a simple risk-management strategy that dynamically varies leverage to target a constant level of volatility. For a given factor, we compute its realized variance each month from returns over the last 21 trading days. Let $\{F_d\}_{d=1}^{D}$ be the daily returns on a factor, F, and $\{d_t\}_{t=1}^{T}$ the time series of the dates of the last trading sessions of each month. Then the realized variance of the factor in month t is:

$$RV_{F,t} = \sum_{j=0}^{20} F_{d_t - j}^2. \tag{3}$$

We use the realized volatility, $\hat{\sigma}_{F,t} = \sqrt{RV_{F,t}}$, to scale the monthly factor returns in order to achieve a given target σ_{target}. Implicitly, $\hat{\sigma}_{F,t}$ is a forecast of the conditional volatility $\sigma_{F,t+1|t}$. The volatility-managed version of the factor is denoted with an asterisk, and its month-t return is defined to be:

$$F_{t+1}^{*} = \frac{\sigma_{target}}{\hat{\sigma}_{F,t}} F_{t+1}. \tag{4}$$

We also refer to volatility management as "volatility timing," "volatility scaling," or "scaling". Barroso and Santa-Clara (2015b) shows that the timing strategy defined by Eq. (4) applied to momentum avoids the pronounced crash risk of the original factor and nearly doubles its Sharpe ratio. This timing approach is simple to construct, avoids hindsight biases documented by Liu, Tang, and Zhou, 2019, and does not produce as extreme weights as the common alternative practice of scaling by realized variance, (e.g., Fleming et al., 2001, 2003; Kirby and Ostdiek, 2012; and Moreira and Muir, 2017).

As in Barroso and Santa-Clara (2015b), we choose σ_{target} corresponding to an annualized volatility of 12%. This choice of target has the desirable property of producing scaled portfolios with approximately the same ex-post volatility of the (unscaled) U.S. stock market portfolio and picking the same target for all portfolios facilitates comparison of performance measures that are sensitive to scaling across factors, such as alphas and standard deviations.

[Insert Table 1 near here]

Table 1 reports descriptive statistics on the performance of FP, LSY, and BAV, along with their respective risk-managed versions. For the chosen volatility target, the scaled factors have approximately the same mean returns with substantially less standard deviation than the original factors. As a result, volatility scaling increases the Sharpe ratios of all three factors by over 40%, although the difference is only significant for the BAB factors (FP and LSY). The economic and statistical significance of these two Sharpe ratio improvements are noteworthy since Cederburg et al. (2020) find that only eight out of 103 managed factors have significantly higher Sharpe ratios than their unmanaged counterparts, and investors would find it difficult to realize benefits implied by other measures of performance, such as alphas, in real time.

Volatility management also reduces higher order risk, resulting in less negative skewness and significantly lower excess kurtosis for all three BAR factors. Favorable improvements in all four return moments caused by volatility scaling lead to significant and economically large increases in certainty-equivalent returns for investors with power utility and constant relative risk aversion (CRRA) of 5.[7] Such investors would pay approximately 6% (LSY*) to 11% (BAV*) per year to trade each of the unmanaged factors for their managed counterparts. The last column shows that volatility management increases the utility of mean-variance investors by over 100% for all three factors.[8] The column labeled 'Alpha' reports intercepts from spanning regressions of the scaled-factor returns on un-scaled factor returns. Alphas range from 2.67% (IVOL) and 4.03% (LSY) per month with corresponding information ratios ranging from 0.28 (IVOL) to 0.52 (LSY).

Overall, the results in Table 1 show that risk-managed BAR strategies outperform their unmanaged counterparts with both statistical and economic significance. These results expand on those documented by prior studies and provide a foundation for our main results in the next section.

3. Leading explanations of the low-risk anomaly

In this section, we examine the consistency of leading theories of the low-risk anomaly with this anomaly's strong performance conditional on low volatility.

[7] We adopt the value of 5 as in Barroso and Santa-Clara (2015a) following the estimated risk-aversion obtained from implied volatility in S&P options in Bliss and Panigirtzoglou (2004).

[8] This measure follows Moreira and Muir (2017) and holds for any level of mean-variance risk aversion.

3.1. The leverage constraints theory

Black (1972) and Frazzini and Pedersen (2014) posit that high-beta stocks have embedded leverage that has economic value for investors facing borrowing constraints. Since they cannot borrow, less risk averse investors will instead invest heavily in high-beta stocks in equilibrium, bidding up their prices and lowering their expected returns relative to the CAPM. Sophisticated financial intermediaries able to leverage and assume the opposite side of the trade (e.g. hedge funds "betting against beta") are exposed to funding risk and margin requirements, which limits their capacity to eliminate the anomaly completely.[9] Frazzini and Pedersen (2014) report a model calibration exercise designed to produce a plausible BAB effect in general equilibrium, although they do not explicitly discuss the predictive relation between volatility and expected returns on BAB. In this section, we identify this predictive relation in their model by comparing several calibrations with different levels of volatility.

In the Frazzini and Pedersen (2014) model, each agent i has initial wealth W_t^i and forms a portfolio of S securities. The number of shares of each security she wants to own is given by the vector $x_i = (x_i^1, ...x_i^S)$. The agent has a quadratic utility function and maximizes the respective objective function:

$$\max_{x_i} x_i'(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t) - \frac{\gamma^i}{2} x_i'\Omega_t x_i, \tag{5}$$

where P_{t+1} and δ_{t+1} are the price and dividend at time $t+1$ (both are S-by-1 vectors), γ^i is a risk aversion parameter, and Ω is the covariance matrix of $P_{t+1} + \delta_{t+1}$. Agents face the restriction that invested wealth cannot exceed the amount required to cover margin requirements:

$$m_t^i \sum_s x_s^i P_t^s \leq \ W_t^i \tag{6}$$

where m_t^i is the margin constraint. An investor that cannot use leverage, for example, has $m_t^i = 1$, meaning that the value of her risky portfolio of assets cannot exceed her wealth. The higher the m_t^i, the tighter becomes the margin requirement. This feature plays a crucial role in the model as investors have different levels of risk aversion and some, the less risk-averse ones, can be constrained by margin requirements, creating a shadow cost of funding constraints.

[9] Consistent with this theory, Adrian, Etula, and Muir (2014), Jylhä (2018), Boguth and Simutin (2018), and Lu and Qin (2021) show that various proxies for the tightness of leverage constraints forecast the returns on betting-against-beta (BAB) factors.

In equilibrium, for any security, the expected return should then be:

$$Er^s_{t+1} = r^f + \psi_t + \gamma \; cov_t(r^s_{t+1}, r^M_{t+1})P'_t x^* \tag{7}$$

where r^s_{t+1}, r^M_{t+1} are, respectively, the security and market returns, ψ_t is an endogenous variable reflecting the aggregate shadow cost of leverage (a function of the average Lagrange multiplier of the constraint in Eq. (6)), and x^* is the S-by-1 vector with the total number of shares outstanding in the economy and that must be owned in equilibrium. Applying equation 7 to the market itself yields:

$$Er^M_{t+1} = r^f + \psi_t + \gamma \; var_t(r^M_{t+1})P'_t x^* \tag{8}$$

Solving Eq. (8) for $P'_t x^*$, plugging into Eq. (7), and rearranging terms yields the main equilibrium relationship of the model:

$$Er^s_{t+1} - r^f = \psi_t(1 - \beta^s_t) + \beta^s_t(Er^M_{t+1} - r^f). \tag{9}$$

In particular, $\psi_t(1 - \beta^s_t)$ is the Jensen's alpha of stock s. As can be seen, as long as some agents are constrained ($\psi_t > 0$), this alpha is negative for high beta stocks ($\beta^s_t > 1$) and positive for low beta stocks.[10]

Eq. (8) highlights a previously neglected feature of the leverage constraints model: a positive market risk-return tradeoff. Because investors are risk averse, the representative risk aversion γ will be positive and the expected return and Sharpe ratio of the market should rise with volatility, counterfactual to the empirical results that the predictive relationship is weak, or even negative, in the data, and the market Sharpe ratio falls with volatility (e.g., Campbell, 1987 and Moreira and Muir, 2017). We next show the model is inconsistent with the performance of the BAB* portfolios as well.

Frazzini and Pedersen (2014) provide the calibration exercise of their model in the Internet Appendix (page B18) for values "chosen to roughly match the empirical volatilities and correlations of the asset returns." They consider an economy with two agents and two assets, which have the same expected payoff of $\frac{1}{2} \times 100/(1 + r^f)$ The payoffs have variances of 40 and 205 and a covariance of 84. The risk-free rate is 3.6% and total wealth split by the two agents is 100. The share of wealth of each agent, their risk aversion parameters, and margin requirements are the only

[10]Note that the argument in the leverage constraints model applies only to beta, not IVOL nor lottery portfolios. These do not produce any risk-adjusted profits, in expectation and in equilibrium, unless their respective characteristics happen to correlate with beta.

exogenous variables needed to fully characterize equilibrium.

[Insert Table 2 about here]

The first three columns of Table 2 replicate baseline calibrations of Frazzini and Pedersen (2014) and the remaining columns show results for different levels of volatility. In the standard CAPM scenario, investors are not constrained and so no BAB effect emerges in equilibrium. But a sizable BAB effect appears when the less risk-averse investor is constrained, which occurs in columns (2) and (3). Frazzini and Pedersen (2014) argue that the results in column (3) are the most suitable to explain the observed BAB effect in the data and adopt it as their baseline calibration, so we focus on that scenario.

In columns (4) through (7), we multiply the covariance matrix of payoffs in the original calibration by a volatility scalar ranging from 0.25 to 2. The expected return of the BAB portfolio monotonically increases from 2%, in the low volatility scenario, to 9% in the high volatility scenario. The increase in expected excess returns is so large that, in spite of higher volatility, the Sharpe ratio of the strategy increases in equilibrium. Both results are the opposite of the empirical results documented in Table 1, where expected returns are either constant or decreasing with volatility and Sharpe ratios fall with volatility.

A second concern is that margin requirements typically increase with volatility. Brokers and central clearing counterparties routinely raise margin requirements on their clients when volatility is high to guard against the elevated likelihood of loss. In column (8), we combine the high volatility scenario in column (7) with higher margin requirements. In this scenario, the BAB Sharpe ratio is even higher than in column (7) since the less risk averse investor becomes even more constrained.

In the Internet Appendix, we show that for alternative configurations of the model, different from those prescribed in the original paper, it is possible to obtain the disappearance of the beta anomaly only for extremely high levels of volatility. Essentially, all agents can become unconstrained when risk is high enough. But these calibrations, again run into the problem of Eq. (8), and generate extreme and implausible scenarios whereby the Sharpe ratio of the market must (steeply) increase with volatility. This, again, contradicts stylized facts on the price of risk falling when volatility increases in the market. We also present several other variants of the baseline calibrations. The most recurring conclusion in all calibrations is that we should observe patterns opposite to what is found in the data for either the Sharpe ratio of BAB or that of the market.

Overall, we conclude that the leverage constraints model cannot explain the variation in BAB Sharpe ratios, or lack of variation in expected returns, conditional on volatility. Intuitively, it is hard to reconcile the economic rationale of the model with the absence of risk-return tradeoffs for both BAB and the market necessary to generate these factors' volatility-timing benefits. In the model, the constrained investors, who are more risk tolerant and therefore cause the beta anomaly, are not unsophisticated, optimistic, or driven by lottery preferences. Rather, they are rational utility-maximizing investors that choose, in equilibrium, to accept low CAPM alphas in high-beta stocks to benefit from their embedded leverage. As risk averse investors, however, they still require larger returns to hold risky investments when volatility increases.

3.2. Limits to arbitrage

Pontiff (2006) argues theoretically that IVOL represents arbitrage risk that is an important holding cost of arbitrage. Consistent with this argument, many studies show that anomaly returns increase in the cross-section with high levels of IVOL (see, e.g., Ali, Hwang, and Trombley, 2003; Mashruwala, Rajgopal, and Shevlin, 2006; Doukas, Kim, and Pantzalis, 2010; Li and Zhang, 2010; and McLean, 2010). Stambaugh et al. (2012) observe that a disproportionate amount of capital exploits underpricing relative to overpricing and show, presumably as a result of this "arbitrage asymmetry," that overpricing is generally more prevalent than underpricing and even more so in times of optimistic sentiment.

Stambaugh et al. (2015) posit that arbitrage risk and asymmetry combined can explain the IVOL puzzle. IVOL deters arbitrage in both underpriced and overpriced stocks, but the effect in overpriced stocks is larger due to the asymmetry. Hence, the effect in overpriced stocks dominates when sorting stocks based on IVOL alone. Consistent with this argument, they show that IVOL is positively related to alpha with respect to the Fama and French (1993) model in underpriced stocks, but the average relation across all stocks is negative. Liu et al. (2018) extend these results and show that the beta anomaly results from the strong positive cross-sectional correlation between beta and IVOL. Specifically, they show that BAB abnormal returns exist only in overpriced stocks and disappear controlling for IVOL.

If the arbitrage asymmetry explanation is consistent with the performance of volatility-managed BAR strategies, then two results should obtain: these factors' abnormal returns should be concentrated in overpriced stocks and, in the case of BAB*, disappear entirely controlling for the cross-sectional effects of IVOL.

3.2.1. Beta, idiosyncratic volatility, and mispricing

To control for the cross-sectional effect of mispricing on BAR anomalies, we use the standard double-sorting approach following Stambaugh et al. (2015), Liu et al. (2018), and Asness et al. (2020). Specifically, each month, we sort stocks into quintiles based on the Stambaugh et al. (2012, 2015) mispricing measure, and then, within each quintile, we sort stocks into five value-weighted portfolios based on their beta or IVOL. We form BAR factors within each mispricing quintile that go long the low-risk portfolio and short the high-risk portfolio. In the case of BAB factors, we scale by portfolio-level ex-ante betas as in Eq. (1). We also construct BAR factors, denoted $\text{BAR}_{\perp}$, that control for the cross-sectional effects of mispricing as the simple average of the BAR factors across the five mispricing quintiles. By construction, the $\text{BAR}_{\perp}$ go long low-risk stocks and short high risk stocks that have similar mispricing in the long and short legs.

[Insert Table 3 about here]

Table 3 presents FF3 alphas for the 25 mispricing-risk portfolios along with differences between these alphas in the top- and bottom-quintile portfolios along both dimensions. To be clear, the portfolios in Table 3 do not use leverage or volatility timing, but create a baseline for subsequent tests. Panels A, B, and C, respectively, report results based on FP betas, LSY betas, and IVOL. Consistent with Liu et al. (2018), the last column of Panels A, B, and C show that the difference in alpha between high- and low-risk stocks is insignificant in the low-mispricing quintile but significantly negative and economically large in the high-mispricing quintile (-1.09% per month in Panel A, -0.56% in Panel B, and -1.68% in Panel C).

Table 4 reports spanning regressions of the volatility-managed BAR factors that control for mispricing on their unmanaged counterparts. By construction, if the abnormal returns of $\text{BAR}^*_{\perp}$ still have performance gains from risk-management, then those gains cannot be attributed to arbitrage asymmetry.

[Insert Table 4 about here]

Panels A and B show no monotonic or otherwise discernible pattern among FP* and LSY* alphas across quintiles. All ten point estimates within mispricing quintiles are positive and six of them are statistically significant at the 5% level. For each beta measure, a bootstrap test shows the difference between alphas in the high- and low-mispricing groups is insignificant. Moreover, both $\text{FP}^*_{\perp}$ and $\text{LSY}^*_{\perp}$ experience robust gains from volatility management, with significant alphas of 0.36% ($\text{FP}^*_{\perp}$) and 0.27%

per month ($\text{LSY}^*_\perp$). Thus, we reject the null that arbitrage asymmetry explains the performance of the BAB portfolios conditional on volatility.

Panel C tells a different story for BAV*, whose alpha is only statistically significant for overpriced stocks. This alpha is noteworthy since it indicates that volatility timing expands, at the margin, on the already strong performance of BAV in these stocks documented in Table 4. The difference in alphas between the lowest and highest quintiles is marginally significant ($t = 1.71$) and $\text{BAV}^*_\perp$ does not demonstrate any significant gain from volatility management, with a small (0.13%) and insignificant ($t = 1.26$) alpha. Overall, the evidence in Panel C shows that the performance of BAV* is consistent with the arbitrage asymmetry explanation of the IVOL anomaly. However, it should be noted that this consistency does not, by itself, provide a complete explanation for this performance, which requires overpriced high-volatility stocks to become even more overpriced relative to low-volatility stocks when the realized variance of BAV decreases. It is also worth noting that the contrast between the results based on betas and those based on IVOL highlights a novel contrast between the two anomalies.

3.2.2. Idiosyncratic volatility and beta

We next investigate whether the performance of BAB* disappears controlling for the cross-sectional effects of IVOL. To do so, we perform a similar double-sorting exercise with IVOL and betas as we did above for mispricing and risk.

[Insert Table 5 about here]

Table 5 reports the FF3 alphas of the 25 IVOL-beta portfolios. The results in both panels are mostly consistent with Liu et al. (2018). The bottom row of each panel shows that the IVOL anomaly is significant in all beta quintiles, with alphas ranging from -0.67% to -1.60% per month. Conversely, Panel B, which uses LSY betas, shows that in four out of five IVOL quintiles, the beta anomaly is statistically insignificant. The exception is quintile 4, where the high-beta portfolio earns a FF3 alpha that is 55 basis points ($t = -2.66$) per month less than that of the low-beta portfolio. The difference of high-minus-low-beta spreads in alphas between volatility quintiles 1 and 5 is also small (-0.19%) and insignificant ($t = -0.19$). However, in Panel A, which uses FP betas, the beta anomaly is statistically significant in all volatility quintiles, although the high-minus-low-beta spread in alphas increases significantly ($t = -2.74$) and monotonically in magnitude from -0.36% in the bottom quintile to -1.16% in the top quintile.

Strictly speaking, the evidence in Panels A and B of Table 5 shows that the IVOL anomaly only subsumes the beta anomaly when using LSY betas. However, this evidence is still consistent with the arbitrage risk interpretation of IVOL since, for both measures of beta, the high-minus-low-beta spreads in alpha increases with IVOL. In contrast, the leverage constraints theory and risk-based explanations do not explain why BAB strategies are more profitable when constructed from high-IVOL stocks.

Next, we examine whether the limits-to-arbitrage interpretation of IVOL affects the volatility-timing benefits of BAB. We build BAB factors within each IVOL quintile from the 25 IVOL-beta portfolios and then average across IVOL quintiles to form factors, denoted "$FP_\perp$" or "$LSY_\perp$", respectively, based on variation in beta that controls for the cross-sectional effect of IVOL. Table 6 presents spanning regressions of the returns on the managed versions of these factors on those of their unscaled counterparts. If limits to arbitrage captured by IVOL drive the BAB anomaly, we would expect the managed BAB strategies should be most profitable in high-IVOL stocks and $BAB^*_\perp$ should not benefit from volatility timing.

[Insert Table 6 about here]

Panel A of Table 6 shows that the managed FP strategies are significant in all five IVOL quintiles with no monotonic pattern across IVOL rankings. Moreover, the $FP^*_\perp$ factor achieves a significant alpha. Similarly, three of the five managed LSY portfolios by IVOL group have significant alphas, with no clear relation between the alphas and IVOL, and the $LSY^*_\perp$ factor has a significant alpha. The alphas of both $BAB^*_\perp$ factors annualize to over 3.7%–3.8% per year, roughly the same as those of the corresponding factors in Table 1 that do not control for IVOL. Overall, the results in Table 6 show that the limits to arbitrage captured by IVOL do not explain the strength of the beta anomaly conditional on low volatility.

3.3. Lottery preferences

The lottery preferences theory of the low-risk anomaly posits that some investors demand high-risk stocks, even if they have low mean returns, because they offer a possibility of very high returns. Consistent with this explanation, several studies find evidence that lottery demand drives down expected returns of "risky" stocks and plays at least a partial role explaining both the beta and (idiosyncratic) volatility anomalies (e.g., Bali, Cakici, and Whitelaw, 2011; Conrad, Kapadia, and Xing, 2014; and Bali et al., 2017). Bali et al. (2011) and Bali et al. (2017) measure lottery demand with a variable called MAX, which is defined for each stock and month as the average of the stock's highest five daily returns over that month. They find that controlling for MAX subsumes the low-risk anomaly. Asness et al. (2020) argue that stocks can have a high MAX either because their returns are volatile or because they are right-skewed. They measure lottery demand with a variable called SMAX, which scales a given stock's MAX by the stock's standard deviation of daily returns in the month MAX is measured. As a result, SMAX captures skewness on a stock while neutralizing the mechanical effect of volatility.

For the performance of the managed BAR results to be consistent with the lottery theory, lottery demand must be lower in high-volatility periods when risky stocks are evidently less overvalued relative to low-risk stocks. But lower demand of lotteries in high-vol states is not consistent with extant facts on lottery preferences. Kumar (2009), and cites therein, all find that lottery demand for a given investor increases during economic downturns when volatility is relatively high. Kumar further argues: "When volatility is high, investors might believe that the extreme return events observed in the past are more likely to be realized again." However, an opposite effect can be at work as high-volatility periods are also "bad times" when "sentiment traders", who may exhibit lottery preferences, tend to exit the market (e.g., Antoniou, Doukas, and Subrahmanyam, 2013; and Antoniou et al., 2016). Thus, increases in volatility can increase demand of risky stocks by lottery-motivated traders that stay in the market, while simultaneously decreasing the number of such traders in the market, rendering the net effect an empirical question.

We test whether lottery demand can explain BAR conditionality using similar methods as in Tables 4 and 6. Specifically, we sort stocks each month into lottery-demand quintiles based on MAX, and then, within each quintile, sort stocks into five risk portfolios. Within each MAX quintile, we form a BAR factor that goes long the low-risk portfolio and shorts the high-risk portfolio, leveraging to maintain beta-neutrality in the BAB factors. Averaging the BAR factors across MAX quintiles generates factors denoted $\text{BAR}_{\perp}$ based on variation in risk that controls for the cross-sectional effects of lottery-demand. We separately form similar factors based on SMAX as well.

[Insert Table 7 about here]

Panels A and B of Table 7 present alphas of the $\text{BAR}^*_{\perp}$ that control for MAX and SMAX, respectively, with respect to their unmanaged counterparts. The alphas are all positive and significant at the 1% level, ranging between 0.27% ($\text{LSY}^*_{\perp}$ and

$BAV^*_\perp$) and 0.42% ($FP^*_\perp$). Moreover, volatility timing significantly increases the Sharpe ratios of all six factors in direct comparisons. So, even BAV*, the strategy with weakest Sharpe ratio gains in our baseline setting of Table 1, shows strong gains after lottery effects are neutralized. Overall, these results show that controlling for lottery demand produces managed BAR factors with stronger performance than in our baseline setting, in particular rejecting the hypothesis that lottery preferences explain the conditional performance of BAR strategies.

3.4. Multifactor explanations of the low-risk anomaly

Novy-Marx and Velikov (2021) show that the FF6 explains the returns on the value-weighted FP strategy due to loadings on profitability, investment, and momentum factors. Schneider et al. (2020) offers a more parsimonious multifactor model for the low-risk anomaly, finding that a premium for coskewness risk accounts for the CAPM alphas of factors based on FP betas and IVOL, albeit over a shorter sample period (1996–2014) than we use. In this subsection, we examine whether these two multifactor models explain the returns on BAR factors conditional on volatility.[11]

[Insert Table 8 about here]

Panel A of Table 8 shows the results for spanning regressions of BAR returns on the realizations of the FF6 factors over different sample periods. The table has three sets of four columns, one set for each BAR portfolio. The first column of each set, labeled 'Unc', presents estimates from the regression using the full sample period, and the second (third) column, labeled 'Low' ('High'), reports a similar regression following months where realized variance is below (above) it median value. The fourth column presents Low-minus-High differences between the second and third columns.

The whole-sample regressions extend the results of Novy-Marx and Velikov (2021) and confirm that all three BAR portfolios earn insignificant alphas. These regressions also show that all three BAR factors load positively on HML, RMW, CMA, and MOM, while the signs of their market and SMB loadings are not the same. LSY and

[11] In the case of the FF6, an important caveat is that variables similar to the new factors in the model, such as profitability, have been used as proxies of mispricing themselves (see, for example, Stambaugh et al., 2015; and Stambaugh and Yuan, 2017). So they can understate the extent of mispricing in the low-risk anomaly. Moreover, Kozak, Nagel, and Santosh (2018) show that asset returns can conform to a factor structure even if the original source of return predictability is due to mispricing. Hence, spanning regressions do not necessarily discriminate between risk-based and (structural) mispricing explanations of the low-risk anomaly. They only assess conformity of the test assets to a factor structure for a given set of factors.

BAV have negative loadings on SMB, while that of FP is positive. It is not surprising that high-IVOL stocks have lower market capitalizations. However, it is perhaps surprising that low-beta stocks in LSY would tend to be large companies, while low-beta stocks in FP would tend to be small companies, and vice versa for high-beta stocks. The zero-cost IVOL portfolio has a large and statistically significant negative market beta, consistent with the fact that high-volatility stocks tend to also have high betas. FP has an insignificant market loading, while that of LSY is significantly positive, although untabulated results show this difference is merely an artifact of the multivariate setting; LSY's market beta in a simple CAPM regression is less than 0.2 and insignificant. Thus, both FP and LSY are approximately market neutral as intended by construction in Eq. (1).

The 'Low' and 'High' columns show that splitting the sample by realized variance uncovers important variation across subsamples. All BAR portfolios earn economically large alphas of 66 to 76 bps per month in low-volatility states. The differences in alphas between low- and high-volatility regimes are all statistically significant and large, ranging from 67bps (BAV) to 144bps (FP). The FF6 alphas of the BAB strategies even flip signs from positive to negative as volatility changes from low to high.

The large increase in abnormal returns of BAR factors going from high- to low-volatility reflects the confluence of two patterns. First, Panel B shows that Sharpe ratios of all BAR strategies more than double from 'High' to 'Low' columns, with increases ranging from 0.36 (BAV) to 0.85 (LSY). These increases are economically large in all cases and statistically significant at 5% level for the two BAB portfolios. Second, the loadings on HML, RMW, CMA, and MOM fall pronouncedly with volatility. For example, the difference between FP and LSY loadings on RMW in 'High' and 'Low' columns are significant and large at -0.89 for FP and -0.92 for LSY. Five of the six loadings of BAV fall significantly going from 'High' to 'Low' columns. Overall, there is a robust "dwindling" effect of BAR loadings when volatility decreases. The R^2 always decrease moving from high- to low-volatility states as well, so the comovement of BAR returns with those of the risk factors is concentrated in the half of the sample where BAR factors earn negative alpha. In the other half, where these factors produces strong alphas and Sharpe ratios, the portfolios show subdued co-movement with risk factors. Taken together, the evidence in Panels A and B challenge risk-based explanations of the low-risk anomaly since their most anomalous returns come exactly when the least risk is required to earn them.

Panel C shows the coskewness betas of each BAR portfolio conditional on realized variance. Following Schneider et al. (2020), to obtain the coskewness betas, we first regress each BAR strategy excess returns on the market factor to obtain the time-series of residuals. Then we regress these residuals on the squared market excess returns in each sample, defining coskewness beta, β_{coskew}, as the slope. The unconditional results for FP confirm those of Schneider et al. (2020) in our longer sample period. The β_{coskew} of FP is -2.62 and statistically significant at the 1% level ($t = -3.03$). Furthermore, the β_{coskew} of FP is larger in magnitude (-3.08) in low-risk periods, however, it is not significantly different than the corresponding quantity in high-risk periods ($t = -0.61$ for the difference). In contrast, we find that the β_{coskew} of LSY and BAV are insignificant over the whole sample, with insignificant differences between 'High' and 'Low' columns. Overall, while our tests lend support to an unconditional coskewness-risk explanation of BAB if defined with FP betas, the time-series variation in this risk does not account for the volatility-timing benefits of FP and the explanation is not robust to using LSY betas or IVOL.[12]

Another multi-factor explanation of the beta anomaly relies on the conditional CAPM of Shanken (1990) and Ferson and Schadt (1996). Conventional tests of the beta anomaly are unconditional while the underlying asset pricing equilibrium relations are conditional. Cederburg and O'Doherty (2016) find that in a conditional CAPM setting, the beta anomaly disappears. Liu et al. (2018) show the Cederburg and O'Doherty (2016) results depend critically on a choice of beta measure that generates a weak spread in CAPM alphas and, in particular, fails using LSY betas. Thus, the conditional CAPM evidently cannot explain the unconditional low-risk anomaly, let alone its volatility puzzle.

4. Extensions and possible explanations

In this section, we investigate the interaction effects of different predictors of BAR returns and present institutional ownership results that potentially help explain the anomalous returns of BAB.

[12] Asness et al. (2020) further argue: "...low-beta stocks have much higher returns than high-beta stocks when the market is down, especially when the market is way down. Hence, [the Schneider et al. (2020)] theory has no chance to explain a flat security market line because low- beta stocks are certainly safer than high-beta stocks with respect to any stochastic discount factor that is monotonic in the market return (as implied by a representative agents with any standard utility function of the market return)."

4.1. Double sorts on sentiment, market volatility, systematic, and specific risk

We perform double sorts of months based on pairs of variables that forecast BAR performance to evaluate their interaction effects and relative importance, which ultimately helps shed light on explanations of the beta and IVOL anomalies. These double sorts split the time periods based on the median of each predictor.

[Insert Table 9 about here]

Antoniou et al. (2016) show that the beta anomaly is nearly absent in pessimistic periods, consistent with mispricing of beta being caused by unsophisticated investors drawn to the market, especially high-beta stocks, in periods of high sentiment. Stambaugh et al. (2015) find that the IVOL anomaly is stronger in times of high-sentiment due to arbitrage asymmetry combined with high valuations driven by excessive optimism. Barroso and Detzel (2021) document that volatility timing the market portfolio is only profitable when sentiment is high, and significantly diminishes performance during pessimistic periods, reflecting the results of Yu and Yuan (2011) that time-series risk-return tradeoff of the market is negative in optimistic periods, but positive otherwise. Motivated by these findings, we compare the Sharpe ratios of the BAR factors across months double-sorted on lagged realized volatility and sentiment in the first three of columns of Table 9. Following Yu and Yuan (2011) and Barroso and Detzel (2021), lagged sentiment is defined as the value of the annual-frequency Baker and Wurgler (2007) sentiment index from the previous year.

The first three columns of Table 9 show that, within each volatility state, Sharpe ratios of the BAR factors are relatively strong when sentiment is optimistic. The one exception is for FP in low-volatility periods, but the difference is relatively small and not statistically significant. Thus, consistent with Antoniou et al. (2016) and Stambaugh et al. (2015), optimism appears to strengthen the BAR anomalies. However, in all comparisons, BAR Sharpe ratios are higher when volatility is low. Hence, sentiment does not explain the volatility-timing benefits of the low-risk anomaly, although we do observe an interaction effect. Pessimistic periods on top of high volatility exhibit abysmal performance, with BAR Sharpe ratios in these states ranging from -0.37 (BAV) to 0.01 (FP).

Barroso and Santa-Clara (2015b) decompose the risk of momentum into a specific and a systematic component using a CAPM regression and find that the specific component drives the benefits of volatility management. We perform a similar exercise in the second set of columns. We decompose the realized variance of each BAR strategy at a monthly frequency by first regressing their daily returns in the month on the realizations of the FF6 factors. Systematic risk for a given month is defined to be the variance of the fitted values from this regressions, and specific risk is defined to be the variance of the corresponding residuals. In line with Barroso and Santa-Clara (2015b), differences across samples split by specific risk (demarcated by columns) are generally higher than across systematic risk (rows). But in most cases, the differences are not statistically significant and no component seems to drive the other out. For all BAR factors, especially BAB, the interaction of the two types of risk reinforces each other. The effects along the diagonal ("High-High" minus "Low-Low") are strong with a t-stat of 2.48, for FP, and 3.29, for LSY. Thus, a combination of high volatility in the two components greatly diminishes the BAR Sharpe ratios.

Eisdorfer and Misirli (2020) propose scaling a healthy-minus-distressed factor with market volatility since it better identifies states of the economy and produces stronger gains than using lagged factor realized volatility. DeMiguel et al. (2021) show that scaling with market volatility performs better than with own-factor volatility. They further argue that using this different conditional variable is enough to overcome the implementation problems of (own-)volatility management based on estimation error and transaction costs documented by Cederburg et al. (2020) and Barroso and Detzel (2021). Thus, we examine the interaction of factor and market volatilities in the third set of columns. We again find a contrast between the beta and IVOL anomalies. For beta, the differences are larger with own-factor realized volatility. For instance, for LSY in periods of high market volatility, the Sharpe ratio decreases from 1.16 to -0.01 when the volatility of the factor increases from low to high ($t = 2.47$). For BAV, market volatility is more important, perhaps in part because it is not designed to be market neutral. While the differences are not statistically significant, the drop in BAV Sharpe ratio from low- to high-market-volatility regimes are large, 0.62 in the 'Low' row and 0.76 in the 'High' row. For both anomalies, though, a combination of high volatilities of both types predicts very weak BAR performance, with Sharpe ratios in these states of -0.01 (LSY) to 0.05 (FP) compared to 0.72 (BAV) to 0.92 (FP) in 'Low'-'Low' states. These differences along the diagonal are all significant at the 5% level.

Overall, we draw two main conclusions from this double-sorting exercise. First, sentiment does not explain the benefits of volatility management. Second, systematic, market, and specific components of volatility all contribute to the volatility-timing benefits of BAR, but own-factor volatility appears strongest for BAB.

4.2. Institutional ownership and betting-against-risk

The results above show that the performance of volatility-managed BAB factors is a puzzle relative to leading explanations of the low-risk anomaly. For this performance to exist, it must be the case that high-beta stocks become less overvalued relative to low-beta stocks when the volatility of these factors increases. This section concludes our analysis by providing evidence on a demand-side force that could provide a proximate cause for this change in relative valuation. Specifically, we examine how institutional ownership of high- and low-beta stocks responds to volatility of the BAB factors. It is well known that institutional ownership has significant price impact and the results from Table 9 suggest that naive sentiment traders are not causing the volatility-timing benefits of the low-risk anomaly (see, e.g, Sias, 1996, Nofsinger and Sias, 1999, Gompers and Metrick, 2001; Deuskar and Johnson, 2011; Gabaix and Koijen, 2021). Several recent studies further show the relationship between institutional ownership and subsequent returns is driven primarily by "short-term" institutions that are highly active and trade most frequently, intuitively because they make large high-impact trades relatively often (see, e.g., Yan and Zhang, 2009; Chichernea, Petkevich, and Zykaj, 2015; Cremers and Pareek, 2015; Cremers, Pareek, and Sautner, 2020).

For each of the extreme beta deciles $q = 1$ (low) or 10 (high), Panels A and B of Table 10 present time-series regressions of portfolio-level institutional ownership, $\%IO_{q,t+1}$, on the natural logarithm of lagged realized variance of the beta factors:

$$\%IO_{q,t+1} = a_q + b_q \cdot \log\left(RV_{BAB,t}\right) + \epsilon_t. \tag{10}$$

$\%IO_{q,t+1}$ is defined to be the value-weighted average across stocks in decile q of the percentage of shares held by all, short-term, or long-term institutions. Following Gompers and Metrick (2001), we use the natural logarithm of realized variance since the units of institutional ownership are percentages, and we remove a simple linear time trend from all variables since institutional ownership experienced a significant upward trend over our sample period.[13] Coefficients are standardized such that b_q in Eq. (10) represents the increase in the percentage of shares owned by institutions corresponding to a one-standard-deviation increase in $\log(RV^2_{BAB})$.

[Insert Table 10 about here]

Panels A and B of Table 10 clearly show that, as BAB volatility increases, short-

[13]Results are robust to using quadratic and square-root time trends as well.

term institutions decrease their positions in high-beta stocks and increase investment in low-beta stocks. Panel A shows that a one standard deviation increase in FP volatility is associated with a 0.62% decrease in ownership of high-beta stocks by short-term institutions and a 0.94% increase in ownership of low-beta stocks, with a significant ($p < 0.01$) difference between these point estimates. Panel B shows that a one standard deviation increase in LSY volatility is associated with a 1.18% decrease in ownership of high-beta stocks by short-term institutions and a 0.22% increase in ownership of low-beta stocks, also with a significant ($p < 0.01$) difference between these slopes. Panel B also shows a similar effect for total institutional ownership and LSY betas, although the coefficients are harder to interpret for longer-term institutions, since, by definition, they trade both less and less proactively. Overall, the findings in Panels A and B show that short-term institutions as a group trade in the wrong direction to benefit from timing the volatility of BAB, which in turn suggests their demand contributes to this anomaly, especially after considering that this group includes the most active arbitrageurs such as hedge funds.[14]

The results in Panels A and B of Table 10 contribute new time-series evidence to a nascent literature that argues widely used performance evaluation contracts contribute to the beta anomaly. Baker et al. (2011) show theoretically that managers judged relative to a unit-beta benchmark like the S&P 500 will overweight high-beta stocks, even if they have negative alpha, because, for a given level of tracking error volatility, they are more likely to beat the benchmark than low-beta stocks due to their high average returns. Consistent with this theory, Christoffersen and Simutin (2017) show that, in the cross-section, mutual fund managers facing the greatest pressure to beat benchmarks tilt their portfolios toward high-beta stocks on average.[15] Performance evaluation contracts reward beating the benchmark, but penalize excessive tracking-error volatility. When BAB volatility increases, the potential contribution of high- and low-beta stocks to tracking error volatility relative to unit-beta indexes should mechanically increase as well since both quantities are driven by the market variance and the residual variance of extreme-beta stocks. Taken together, while managers prefer to unconditionally overweight high-beta stocks, an increase in BAB volatility encourages them to shift their portfolio weights towards those of the

[14]Cremers and Pareek (2015) find similarly, that short-term institutions tend to trade in the wrong direction to benefit from momentum and reversal, suggesting their demand contributes to these anomalies in the first place or that they are poor at trading relative to several of the most famous anomalies.

[15]Boguth and Simutin (2018) also shows that active mutual funds have betas that are greater than one on average.

benchmark. This shift requires selling high-beta stocks and buying low-beta stocks as BAB volatility rises, consistent with the results from Panels A and B of Table 10 for short-term institutions.

Panel C of Table 10 provides analogous results as Panels A and B, but for portfolios based on IVOL instead of beta. Overall, the evidence in this panel is weak and inconclusive and therefore does not provide unassailable inferences. However, this null result is consistent with our findings above that suggest the IVOL and beta anomalies are driven by different forces. It is also consistent with the benchmark-incentives interpretation of the evidence in Panels A and B, because, unlike high-beta stocks, high-IVOL stocks have very low expected returns and are therefore useless to managers trying to beat unit-beta benchmarks (see, e.g., Ang et al., 2006).

Overall, the evidence in Table 10 is consistent with institutional demands contributing to the volatility puzzle of the beta anomaly. This pattern correlates naturally with recent theory and evidence that show performance evaluation relative to benchmarks incentivizes asset managers to hold high-beta stocks. While the null results for institutional trading of IVOL portfolios are expected given benchmark incentives, they also leave the limits-to-arbitrage theory as the leading explanation of the low-IVOL anomaly.

5. Conclusion

For fifty years, the finance literature has struggled to explain the puzzling weak cross-sectional relationship between risk and return, which challenges foundational principals of asset pricing. This literature posits a wide range of explanations for the anomaly. Perhaps the earliest of them, the leverage constraints theory argues that high-beta stocks appear overvalued because they give relatively risk-tolerant investors the chance to earn high returns without borrowing. More recently, some argue that investors value the lottery-like payoffs of high-risk stocks or are naively optimistic about their prospects. Others argue that limits to arbitrage allow this anomaly to persist or that it can be explained by missing risk factors.

These explanations all have empirical backing, however, most prior studies do not rigorously test them in a conditional setting. The abnormal returns and Sharpe ratios of betting-against-risk factors based on the anomaly rise dramatically when the ex ante volatility of these factors is low. This leads us to inquire which explanations can simultaneously match the cross-sectional risk-return patterns and this new element of time-series predictability. The resulting exercises show that this dual achievement is more demanding than what the current state of theory can offer, at least for the beta-based definition of the low-risk anomaly.

We show that the leverage constraints theory counterfactually predicts a positive relationship between volatility and subsequent Sharpe ratios of beta factors, intuitively because it depends critically on the assumption of risk averse investors who naturally demand a positive risk-return tradeoff in both the cross-section and time-series. Empirically, we find that beta factors that control for the cross-sectional effects of lottery preferences and limits to arbitrage exhibit the same strong performance conditional on low volatility as their counterparts that ignore these effects. In contrast, we find that the limits-to-arbitrage explanation appears to be consistent with the volatility-based definition of the anomaly. Idiosyncratic volatility factors that control for the cross-sectional effects of mispricing do not exhibit significant performance gains from volatility timing. The contrast in findings between the beta and idiosyncratic volatility anomalies also suggests they are distinct phenomena, contrary to the hypothesis that former is an artifact of the latter. Further results show that multifactor asset-pricing models fail to explain the conditional returns of betting-against-risk factors. The loadings that explain their returns unconditionally generally shrink or flip signs when volatility falls. Said differently, the betting-against-risk factors earn the most anomalous returns precisely when they have the lowest risk.

The results in this paper highlight serious problems associated with ignoring conditioning information in asset-pricing research. Five decades worth of explanations for the beta anomaly uniformly fail to explain its performance conditional on volatility. While many studies estimate conditional asset-pricing models, these studies are generally the exception, and not the norm, with unconditional tests still dominating the literature.

Towards an explanation of our findings, we show that highly active institutions, who are known to have strong price impact, sell high-beta stocks, and buy low-beta stocks, when the volatility of beta factors increases. This trading pattern is consistent with institutional demand decreasing the "overvaluation" of high-beta stocks relative to those with low betas when volatility increases, which is a necessary condition for the performance of beta factors to deteriorate after realizations of high risk. Incentives from widely used benchmark-based performance-evaluation contracts can explain these trading patterns, though future work is needed to fully rule out alternatives.

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Table 1: Performance of managed betting-against-risk portfolios

At the beginning of each month, we sort all stocks into value-weighted decile portfolios based on one of two beta measures or idiosyncratic volatility (IVOL). From these sorts, we form three betting-against-risk (BAR) factors that go long the lowest-risk portfolio and short the highest-risk portfolio. When sorting on beta, we scale each leg of the factors by the inverse of their value-weighted beta at the time of portfolio formation to make the factor market neutral. The risk measures are the beta of Frazzini and Pedersen (2014), the beta of Liu et al. (2018), and IVOL relative to the Fama and French (1993) model (FF3) based on daily data over the previous month. BAR factors based on these risk measures are denoted FP, LSY, and BAV, respectively. This table presents descriptive statistics of the excess returns of the BAR factors and their risk-managed versions (BAR*), which all target 12% annualized volatility by scaling the unmanaged factors proportionally to their inverse realized volatility, which is estimated with daily data over the previous month. The statistics shown are: mean excess return (Mean); standard deviation (STD); skewness (Skew); excess kurtosis (Kurt); Sharpe ratio (SR); certainty equivalent return (CE(5)); alpha of the managed factor with respect to the unmanaged factor (Alpha); information ratio (IR); and percentage gain in mean-variance utility from volatility management ($\Delta U_{MV}(\%)$). Mean, STD, SR, CE, and alpha are all annualized and in units of percentage points. CE(5) is for a power utility function with CRRA of 5. $\Delta U_{MV}(\%) = (SR^2(BAR, BAR^*) - SR^2(BAR))/(SR^2(BAR))$, where $SR^2(F)$ denotes the maximum Sharpe ratio attainable from a set of factors F. The sample period is July 1967 to December 2016. Parentheses below statistics contain p-values for tests of the differences of that statistic between the managed factors and their unmanaged counterparts based on 100,000 bootstrap samples drawn with replacement.

	Mean	STD	Skew	Kurt	SR	CE(5)	Alpha	IR	$\Delta U_{MV}(\%)$
FP	10.94	25.47	-0.19	2.72	0.43	-7.02%	-	-	-
FP*	9.22	14.71	-0.05	0.98	0.63	3.76%	3.80%	0.50	135.98%
			(0.52)	(0.00)	(0.00)	(0.00)	(0.00)		
LSY	8.40	19.84	-0.88	6.18	0.42	-3.84%	-	-	-
LSY*	9.96	16.03	0.24	1.63	0.62	3.62%	4.03%	0.52	149.62%
			(0.02)	(0.02)	(0.00)	(0.00)	(0.00)		
BAV	6.58	24.21	-0.61	6.38	0.27	-12.76%	-	-	-
BAV*	6.81	18.04	-0.20	1.34	0.38	-1.66%	2.67%	0.28	103.43%
			(0.36)	(0.00)	(0.12)	(0.00)	(0.06)		

Table 2: Calibrations of the Frazzini and Pedersen (2014) leverage constraints model

In the model, two agents with different relative risk aversion, shares of wealth, and leverage constraints (m1 and m2) trade a pair of risky assets that together comprise the market. The first column shows a baseline calibration in which leverage constraints do not bind and the CAPM holds. The following two columns replicate scenarios with leverage constraints from Frazzini and Pedersen (2014). Columns (4) to (7) present calibrations that each multiply the baseline covariance matrix of asset payoffs by a "volatility scalar" ranging from 0.25 to 2 (see Section 3.1 for details). The last column shows the combined effects of high volatility with high margin requirements. The first six rows specify the exogenous variables. Remaining rows specify the endogenous outcome variables, which include the annual volatility, expected excess return, Sharpe ratio, and beta of, the low-risk (L) asset, the high-risk asset (H), the market portfolio (MKT), and the betting-against-beta (BAB) factor. The last row contains the shadow cost of leverage constraints for the less risk averse investor (ψ).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Exogenous variables								
Volatility scalar	1	1	1	0.25	0.5	1.5	2	2
Risk aversion 1	1	1	1	1	1	1	1	1
Risk aversion 2	1	10	10	10	10	10	10	10
Wealth share 1	0.5	0.5	0.8	0.8	0.8	0.8	0.8	0.8
m1	1	1	1.2	1.2	1.2	1.2	1.2	1.5
m2	1	1	0	0	0	0	0	0
Endogenous variables								
Vol L	13%	14%	14%	7%	9%	18%	21%	22%
Vol H	30%	33%	33%	15%	22%	42%	51%	55%
Vol MKT	21%	23%	23%	11%	15%	29%	35%	37%
Vol BAB	8%	9%	9%	4%	6%	11%	14%	15%
Excess return L	3%	9%	10%	3%	6%	14%	17%	26%
Excess return H	6%	16%	15%	4%	8%	22%	28%	37%
Excess return MKT	4%	12%	13%	3%	7%	18%	22%	31%
Excess return BAB	0%	4%	7%	2%	4%	9%	11%	19%
SR MKT	0.20	0.55	0.55	0.33	0.44	0.61	0.64	0.84
SR BAB	0.00	0.47	0.78	0.51	0.66	0.80	0.78	1.31
β_L	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
β_H	1.4	1.4	1.4	1.4	1.4	1.4	1.5	1.5
ψ	0%	4%	7%	2%	4%	9%	10%	19%

Table 3: Abnormal returns of portfolios sorted on mispricing and risk

This table presents alphas with respect to the Fama and French (1993) three-factor model (FF3) of 25 value-weighted portfolios formed at the beginning of each month by sorting stocks into quintiles based on the mispricing measure of Stambaugh et al. (2015), and then within each quintile, into five portfolios based on beta or IVOL, as specified by the panel heading. In each panel, the last column (row) shows high-minus-low differences in alphas between the high- and low-risk (mispricing) portfolios. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is from July 1967 to December 2016.

Mispricing	Risk quintile					High-
quintile	Low	2	3	4	High	Low
Panel A: Frazzini and Pedersen (2014) betas						
Low	0.40	0.27	0.25	0.11	0.13	-0.26
	(3.37)	(2.93)	(2.73)	(1.22)	(1.26)	(-1.53)
2	0.33	0.23	0.18	-0.11	-0.26	-0.60
	(2.86)	(2.2)	(2.03)	(-1.21)	(-2.29)	(-3.12)
3	0.24	0.07	-0.06	-0.11	-0.14	-0.38
	(2.00)	(0.66)	(-0.64)	(-1.05)	(-1.12)	(-1.95)
4	0.10	0.13	-0.11	-0.42	-0.72	-0.82
	(0.87)	(1.26)	(-1.07)	(-4.03)	(-5.67)	(-4.33)
High	-0.26	-0.19	-0.59	-0.73	-1.35	-1.09
	(-2.11)	(-1.68)	(-4.77)	(-5.52)	(-7.38)	(-4.54)
High-	-0.66	-0.46	-0.83	-0.84	-1.48	-0.82
Low	(-4.41)	(-3.63)	(-5.70)	(-5.00)	(-7.24)	(-3.48)
Panel B: Liu et al. (2018) betas						
Low	0.28	0.29	0.14	0.28	0.27	-0.01
	(2.78)	(3.09)	(1.51)	(2.79)	(2.25)	(-0.05)
2	0.17	0.12	0.12	-0.01	-0.13	-0.30
	(1.53)	(1.39)	(1.29)	(-0.16)	(-1.13)	(-1.79)
3	-0.04	0.01	-0.01	-0.15	0.05	0.08
	(-0.32)	(0.15)	(-0.06)	(-1.61)	(0.40)	(0.44)
4	0.03	-0.21	-0.49	-0.29	-0.36	-0.39
	(0.21)	(-2.05)	(-4.66)	(-2.43)	(-2.94)	(-2.05)
High	-0.36	-0.47	-0.71	-0.70	-0.93	-0.56
	(-2.85)	(-3.74)	(-5.04)	(-5.07)	(-6.00)	(-2.76)
High-	-0.64	-0.76	-0.85	-0.98	-1.20	-0.55
Low	(-4.46)	(-4.51)	(-4.71)	(-5.37)	(-6.57)	(-2.57)

Table 3: Continued

Panel C: Idiosyncratic volatility						
Mispricing	Risk quintile					High-
quintile	Low	2	3	4	High	Low
Low	0.13	0.29	0.42	0.45	0.40	0.26
	(1.81)	(3.85)	(4.65)	(4.2)	(2.87)	(1.56)
2	0.04	0.23	0.09	-0.03	-0.09	-0.13
	(0.56)	(3.14)	(0.96)	(-0.28)	(-0.6)	(-0.71)
3	0.06	-0.15	0.00	-0.10	-0.22	-0.28
	(0.78)	(-1.79)	(0.03)	(-0.85)	(-1.42)	(-1.49)
4	-0.08	-0.24	-0.20	-0.34	-0.88	-0.79
	(-0.96)	(-2.55)	(-1.78)	(-2.57)	(-5.54)	(-4.24)
High	-0.39	-0.67	-0.83	-1.24	-2.06	-1.68
	(-3.64)	(-4.85)	(-5.95)	(-8.30)	(-10.73)	(-8.15)
High-	-0.52	-0.97	-1.26	-1.69	-2.46	-1.94
Low	(-3.70)	(-5.76)	(-6.99)	(-8.61)	(-10.64)	(-7.87)

Table 4: Performance of volatility-managed betting-against-risk factors controlling for mispricing

Using the 25 mispricing-risk portfolios from Table 3, we make BAR factors within each mispricing quintile that go long that quintile's low-risk portfolio and short the corresponding high-risk portfolio. Factors based on beta leverage the long and short legs by the inverse of their ex-ante beta. BAR factors that control for mispricing, denoted $BAR_{\perp}$, are defined as the simple average of the five factors across quintiles. This table presents spanning regressions of returns on volatility-managed versions of the six factors on those of their unmanaged counterparts. Each panel uses a different proxy for risk. Columns labeled 'Alpha' report regression intercepts. The t-statistics for the High-minus-Low differences in alphas use bootstrap standard errors based on 100,000 simulated samples. All other t-statistics use White (1980) heteroskedasticity-robust standard errors. The sample period is July 1967 to December 2016.

Mispricing quintile	Alpha	t-stat	Slope	t-stat	R^2
Panel A: Frazzini and Pedersen (2014) betas					
Low	0.18	2.50	0.67	27.69	80.79
2	0.37	3.90	0.64	17.01	72.37
3	0.20	2.32	0.61	17.57	74.32
4	0.17	1.96	0.63	18.50	75.37
High	0.27	3.27	0.66	23.72	78.20
High-Low	0.09	0.93	-	-	-
$FP_{\perp}$	0.36	3.80	0.77	17.44	72.13
Panel B: Liu et al. (2018) betas					
Low	0.23	2.99	0.62	16.04	80.05
2	0.27	3.08	0.63	16.33	77.27
3	0.15	1.65	0.60	15.77	75.27
4	0.10	1.09	0.64	19.05	76.07
High	0.17	1.47	0.57	13.25	68.15
High-Low	-0.06	-0.50	-	-	-
$LSY_{\perp}$	0.27	2.41	0.77	14.40	69.80
Panel C: Idiosyncratic volatility					
Low	0.08	0.97	0.70	9.86	77.79
2	-0.03	-0.36	0.66	16.36	78.38
3	0.01	0.07	0.59	14.55	75.81
4	0.16	1.70	0.60	16.61	74.48
High	0.27	2.76	0.52	15.59	73.95
High-Low	0.19	1.71	-	-	-
$BAV_{\perp}$	0.13	1.26	0.77	12.27	71.95

Table 5: Abnormal returns of portfolios sorted on idiosyncratic volatility and beta

This table presents FF3 alphas of 25 value-weighted portfolios formed at the beginning of each month by sorting stocks into quintiles based on IVOL, and then within each quintile, into five portfolios based on the beta measure specified by the panel heading. In each panel, the last column (row) shows high-minus-low differences in alphas between the high- and low-beta (IVOL) portfolios. Parentheses below point estimates contain *t*-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is from July 1967 to December 2016.

IVOL	Beta quintile					High-
quintile	Low	2	3	4	High	Low
Panel A: Frazzini and Pedersen (2014) betas						
Low	0.25	0.17	0.07	0.11	-0.11	-0.36
	(2.32)	(1.85)	(0.82)	(1.37)	(-1.35)	(-2.42)
2	0.16	0.29	-0.01	-0.02	-0.24	-0.40
	(1.41)	(3.15)	(-0.08)	(-0.19)	(-2.33)	(-2.36)
3	0.22	0.23	0.01	-0.13	-0.30	-0.52
	(1.77)	(2.37)	(0.11)	(-1.26)	(-2.37)	(-2.70)
4	0.17	0.12	-0.18	-0.28	-0.85	-1.02
	(1.25)	(0.96)	(-1.43)	(-2.26)	(-4.94)	(-4.43)
High	-0.55	-0.53	-0.75	-0.86	-1.71	-1.16
	(-3.06)	(-3.23)	(-4.06)	(-4.39)	(-7.43)	(-3.91)
High-	-0.81	-0.70	-0.82	-0.97	-1.60	-0.80
Low	(-4.03)	(-3.83)	(-3.98)	(-4.29)	(-6.60)	(-2.74)
Panel B: Liu et al. (2018) betas						
Low	0.14	0.19	0.14	-0.09	-0.07	-0.21
	(1.56)	(2.38)	(1.75)	(-1.25)	(-0.76)	(-1.55)
2	0.15	0.06	0.01	-0.12	-0.15	-0.30
	(1.56)	(0.71)	(0.12)	(-1.45)	(-1.48)	(-1.92)
3	0.03	-0.05	0.12	-0.16	-0.22	-0.25
	(0.28)	(-0.44)	(0.93)	(-1.40)	(-1.66)	(-1.29)
4	0.05	-0.11	-0.18	-0.22	-0.50	-0.55
	(0.40)	(-0.83)	(-1.22)	(-1.37)	(-3.27)	(-2.66)
High	-0.75	-0.89	-0.53	-1.30	-1.15	-0.40
	(-3.98)	(-4.26)	(-2.40)	(-5.90)	(-4.97)	(-1.35)
High-	-0.89	-1.07	-0.67	-1.21	-1.08	-0.19
Low	(-4.26)	(-4.84)	(-2.73)	(-4.83)	(-4.37)	(-0.64)

Table 6: Performance of volatility-managed betting-against-beta factors controlling for idiosyncratic volatility

Using the 25 IVOL-beta portfolios from Table 5, we make BAB factors within each IVOL quintile that go long that quintile's low-beta portfolio and short the corresponding high-beta portfolio, leveraging the long and short legs by the inverse of their ex-ante beta. Factors that control for IVOL, denoted $\text{BAR}_{\perp}$, are defined as the simple average of the five factors across quintiles. This table presents spanning regressions of the returns on volatility-managed versions of the BAB factors on those of their unmanaged counterparts. Each panel uses a different proxy for risk. Columns labeled 'Alpha' report regression intercepts. The t-statistics for the High-minus-Low differences in alphas use bootstrap standard errors based on 100,000 simulated samples. All other t-statistics use White (1980) heteroskedasticity-robust standard errors. The sample period is July 1967 to December 2016.

IVOL quintile	Alpha	t-stat	Slope	t-stat	R^2
Panel A: Frazzini and Pedersen (2014) betas					
Low	0.20	(2.35)	0.75	(21.64)	78.59
2	0.21	(2.79)	0.66	(23.25)	79.21
3	0.25	(3.62)	0.59	(24.56)	79.29
4	0.33	(4.49)	0.47	(13.22)	73.35
High	0.21	(2.67)	0.44	(19.98)	77.52
High-Low	0.01	(0.05)			
$\text{FP}_{\perp}$	0.31	(4.15)	0.79	(19.47)	77.72
Panel B: Liu et al. (2018) betas					
Low	0.10	(1.49)	0.75	(28.25)	84.79
2	0.20	(2.34)	0.63	(17.49)	73.96
3	0.19	(2.15)	0.58	(17.20)	74.25
4	0.24	(3.11)	0.54	(17.40)	74.17
High	0.06	(0.73)	0.47	(19.80)	78.09
High-Low	-0.05	(-0.46)			
$\text{LSY}_{\perp}$	0.32	(3.12)	0.83	(16.68)	71.64

Table 7: Performance of volatility-managed betting-against-risk factors controlling for lottery demand

At the beginning of each month, we sorts stocks into quintiles based on a measure of lottery demand, and then within each quintile, into five portfolios based on beta or IVOL. Using these 25 portfolios, we form BAR factors within each lottery quintile that go long that quintile's low-risk portfolio and short the corresponding high-risk portfolio. In the case of beta factors, we leverage the long and short legs by the inverse of their ex-ante beta. Factors that control for lottery effects, denoted $\text{BAR}_{\perp}$, are defined as the simple average of the five factors across lottery quintiles. This table presents spanning regressions of the returns on volatility-managed versions of these factors, denoted $\text{BAR}^*_{\perp}$, on those of their unmanaged counterparts. Specifications labeled FP, LSY, and BAV, respectively, correspond to portfolios based on Frazzini and Pedersen (2014) beta, Liu et al. (2018) beta, and IVOL, respectively. In Panel A, the lottery demand measure is MAX, which is the average of a stock's highest five daily returns over the prior month. In Panel B, the measure is SMAX, which divides MAX by the standard deviation of the stock's daily returns over the same month. The first six rows contain regression slopes. The row labeled 'Alpha' contains intercepts (% per month). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. Beneath the regression statistics are annualized Sharpe ratios of the unmanaged factors, $SR(BAR_{\perp})$, and the managed factors, $SR(BAR^*_{\perp})$, along with Jobson and Korkie (1981) p-values, $p(\Delta SR)$, for the difference between the two ratios. The sample period is July 1967 to December 2016.

	Panel A: MAX			Panel B: SMAX		
	$\text{FP}^*_{\perp}$	$\text{LSY}^*_{\perp}$	$\text{BAV}^*_{\perp}$	$\text{FP}^*_{\perp}$	$\text{LSY}^*_{\perp}$	$\text{BAV}^*_{\perp}$
$\text{FP}_{\perp}$	0.87			0.80		
	(20.74)			(17.64)		
$\text{LSY}_{\perp}$		0.82			0.86	
		(14.79)			(12.41)	
$\text{BAV}_{\perp}$			1.25			0.75
			(20.72)			(15.46)
Alpha	0.35	0.27	0.27	0.42	0.34	0.36
	(4.56)	(2.70)	(2.94)	(4.15)	(2.78)	(2.80)
R^2	78.33	70.91	80.96	72.20	68.34	74.94
$SR(BAR_{\perp})$	0.49	0.26	0.27	0.52	0.40	0.20
$SR(BAR^*_{\perp})$	0.72	0.44	0.43	0.76	0.58	0.38
$p(\Delta SR)$	0.001	0.024	0.011	0.003	0.032	0.013

Table 8: Multifactor explanations of the low-risk anomaly conditional on volatility

Panel A presents spanning regressions of the returns on BAR strategies (defined in Table 1) on those of the factors from the Fama and French (2018) six-factor model (FF6) conditional on lagged realized BAR volatility. Each set of four columns has results for one BAR portfolio: an unconditional regression over the full sample ('Unc.'), one using the subsample of months with below-median lagged volatility ('Low'), one using the subsample period with above-median volatility ('High'), and the corresponding Low-minus-High differences ('L-H'). Parentheses below point estimates contain t-statistics that use White (1980) heteroskedasticity-robust standard errors. Panels B and C show the Sharpe ratios (SR) and coskewness betas (β_{coskew}), respectively, of the BAR factors over the subsamples specified by the column headings. The bottom row of Panel B contains p-values ($p(SR)$) for the Low-minus-High differences in Sharpe ratios based on 100,000 bootstrap samples. The sample period is July 1967 to December 2016.

	FP				LSY				BAV			
	Unc.	Low	High	L-H	Unc.	Low	High	L-H	Unc.	Low	High	L-H
	Panel A : Fama and French (2018) six-factor model model											
Alpha	-0.07	0.76	-0.67	1.44	-0.17	0.66	-0.47	1.13	0.16	0.72	0.04	0.67
	(-0.22)	(2.57)	(-1.23)	(2.31)	(-0.68)	(2.69)	(-1.24)	(2.50)	(0.95)	(4.47)	(0.17)	(2.17)
RMRF	0.09	0.08	0.12	-0.04	0.57	0.43	0.66	-0.23	-0.37	-0.23	-0.46	0.22
	(1.12)	(1.01)	(0.92)	(-0.28)	(8.32)	(6.29)	(6.39)	(-1.81)	(-9.12)	(-5.68)	(-7.57)	(3.08)
SMB	0.40	0.18	0.52	-0.33	-0.57	-0.61	-0.56	-0.05	-0.99	-1.20	-0.91	-0.29
	(3.99)	(1.53)	(3.48)	(-1.75)	(-5.44)	(-6.77)	(-3.58)	(-0.3)	(-14.55)	(-18.66)	(-9.50)	(-2.48)
HML	0.60	0.49	0.56	-0.08	0.27	0.13	0.25	-0.12	0.21	0.24	0.28	-0.03
	(3.65)	(2.52)	(2.36)	(-0.25)	(2.44)	(0.92)	(1.64)	(-0.56)	(2.58)	(2.63)	(2.61)	(-0.24)
RMW	0.70	-0.02	0.87	-0.89	0.60	-0.13	0.79	-0.92	1.01	0.57	1.03	-0.47
	(4.6)	(-0.08)	(4.44)	(-2.99)	(4.72)	(-0.77)	(4.84)	(-3.91)	(8.68)	(5.2)	(7.67)	(-2.70)
CMA	0.68	0.29	0.87	-0.58	0.79	0.55	0.85	-0.30	0.75	0.29	0.75	-0.46
	(2.86)	(0.99)	(2.52)	(-1.27)	(5.09)	(2.39)	(4.27)	(-0.98)	(5.87)	(2.28)	(4.58)	(-2.20)
MOM	0.30	0.21	0.32	-0.11	0.25	0.06	0.31	-0.25	0.26	-0.05	0.32	-0.37
	(3.21)	(2.00)	(2.68)	(-0.67)	(2.78)	(0.75)	(2.77)	(-1.84)	(2.63)	(-0.65)	(2.77)	(-2.69)
R^2	19.43	9.68	22.90	-	33.09	24.28	38.41	-	75.28	73.71	78.09	-
	Panel B: Sharpe ratios											
SR	0.43	0.87	0.24	0.64	0.42	0.97	0.12	0.85	0.27	0.53	0.17	0.36
$p(SR)$	-	-	-	0.02	-	-	-	0.00	-	-	-	0.23
	Panel C: Coskewness betas											
β_{coskew}	-2.62	-3.08	-1.90	-1.18	-0.53	-1.57	-0.12	-1.45	0.27	0.16	0.42	-0.26
	(-3.03)	(-5.3)	(-1.04)	(-0.61)	(-0.61)	(-1.69)	(-0.11)	(-1.04)	(0.32)	(0.16)	(0.31)	(-0.15)

Table 9: Double-sorted subsamples

For a given predictor variable, we classify each month in our sample period as "High" or "Low" based on the whether the variable is above or below its sample median value, respectively. For each of three pairs of conditioning variables, this table presents comparisons of annualized BAR Sharpe ratios across the four subsample periods defined by the intersection of High and Low classifications for the two variables in the pair. Each set of three columns corresponds to a pair of predictors, with Low and High values of the first (second) variable in the pair specified by the row (column) heading. The first set of three columns sorts on realized BAR volatility and the Baker and Wurgler (2007) sentiment index. "Opt." stands for optimistic (High) sentiment and "Pess." for pessimistic (Low). The second set sorts on systematic and specific risk relative to the FF6 model estimated using daily returns in the previous month. The third set sorts on BAR realized volatility and the realized volatility of the market factor. Each panel shows the results for a different BAR factor (defined in Table 1). Each 3x3 set of cells shows the 2x2 Sharpe ratios and the t-statistic for the test in the difference along the respective row or column. The bottom-right corner of each set presents a t-statistic for the difference in Sharpe ratios along the diagonal. All t-statistics are based on 100,000 bootstrap samples. The sample period is July 1967 to December 2016.

	BAR RV vs. Sentiment			Systematic vs. Specific			BAR RV vs. Market RV		
	Opt.	Pess.	t-stat	Low	High	t-stat	Low	High	t-stat
Panel A: FP									
Low	0.73	0.95	(-0.50)	1.07	-0.07	(2.36)	0.92	0.84	(0.16)
High	0.37	0.01	(0.88)	1.06	0.23	(1.54)	0.63	0.03	(1.41)
t-stat	(0.86)	(2.21)	(1.62)	(0.03)	(-0.61)	(2.48)	(0.64)	(1.84)	(2.37)
Panel B: LSY									
Low	1.22	0.69	(1.28)	1.05	0.69	(0.80)	0.89	1.16	(-0.56)
High	0.25	-0.12	(0.89)	0.64	-0.08	(1.41)	0.43	-0.01	(0.90)
t-stat	(2.34)	(1.96)	(3.13)	(0.82)	(1.72)	(3.29)	(0.95)	(2.47)	(2.69)
Panel C: BAV									
Low	1.05	-0.06	(2.69)	0.72	0.14	(1.30)	0.72	0.10	(1.36)
High	0.64	-0.37	(2.28)	0.37	0.10	(0.59)	0.81	0.05	(1.71)
t-stat	(0.97)	(0.81)	(3.75)	(0.75)	(0.08)	(1.83)	(-0.19)	(0.12)	(2.05)

Table 10: Time-series regressions of institutional ownership of high- and low-risk stocks on realized volatility of betting-against-risk factors

This table presents slopes from regressions of the form:

$$\%IO_{r,q,t+1} = a_q + b_q \cdot \log\left(RV_{\mathrm{BAR},t}\right) + \epsilon_t,$$

where $\%IO_{r,q,t}$ denotes the value-weighted average of the percentage institutional ownership of stocks in decile q of risk measure r at the end of quarter t and $RV_{\mathrm{BAR},t}$ is the realized variance of the corresponding BAR factor. Each panel specifies a different risk measure. The three sequential pairs of columns correspond, respectively, to measuring $\%IO$ as total, short-term, and long-term institutional ownership. The $\log(RV)$ is standardized so that a coefficient of b_q indicates the change in $\%IO$ corresponding to a one-standard-deviation increase in $\log(RV_{\mathrm{BAR}})$. Beneath each pair is a two-sided p value for the null hypothesis that $b_{Low} = b_{High}$. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is quarter 2 of 1980 through quarter 4 of 2016 ($N = 147$).

	Total IO		Short-term IO		Long-term IO	
	Low risk	High risk	Low risk	High risk	Low risk	High risk
Panel A: Frazzini and Pedersen (2014) betas						
b	1.49	0.55	0.94	-0.62	-0.61	-0.17
	(2.02)	(1.27)	(7.92)	(-2.86)	(-1.76)	(-0.73)
$p(b_{High} - b_{Low})$		0.281		0.000		0.327
Panel B: Liu et al. (2018) betas						
b	1.04	-0.94	0.22	-1.18	-0.85	-0.57
	(3.40)	(-2.11)	(2.44)	(-6.13)	(-5.71)	(-2.75)
$p(b_{High} - b_{Low})$		0.000		0.000		0.112
Panel C: IVOL						
b	-0.54	-0.62	-0.06	-0.19	-1.39	-0.79
	(-2.11)	(-0.98)	(-0.72)	(-0.91)	(-8.21)	(-2.74)
$p(b_{High} - b_{Low})$		0.911		0.628		0.044